



MIT 2.S01 Introduction to  
Autonomous Underwater  
Vehicles

# Lecture 8: Hydrostatics

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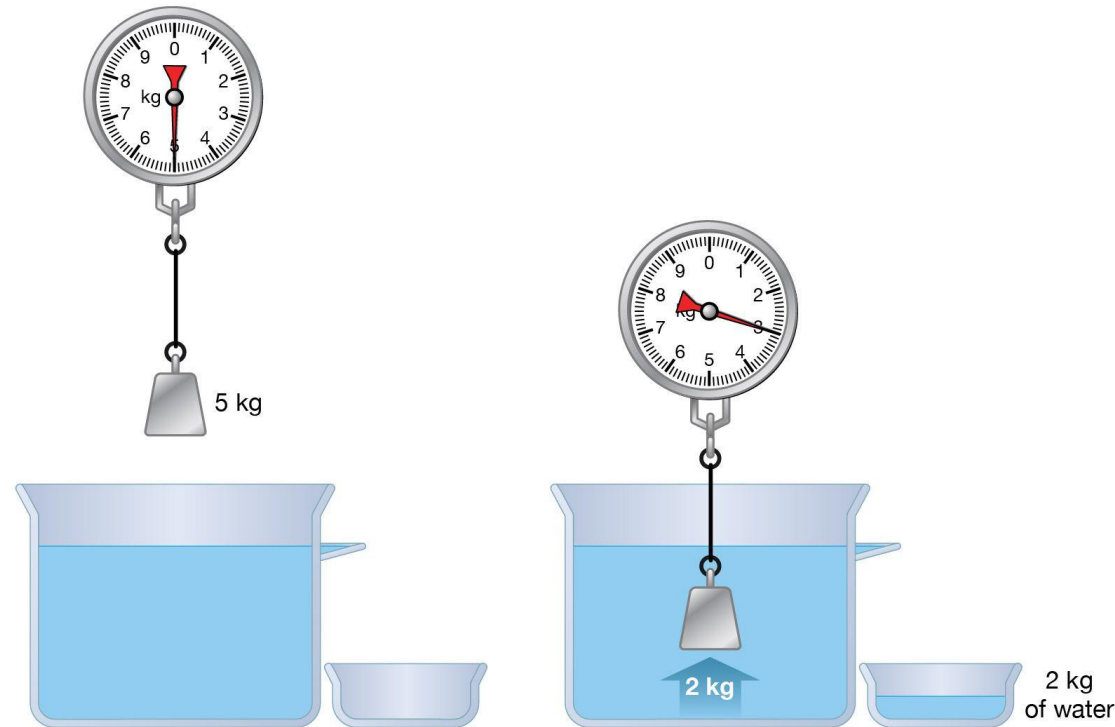


# Archimedes' Principle

“An arbitrary shaped body immersed, either partly or fully, in a fluid will experience the effect of a net positive vertical force originating from the fluid pressure (depth dependent). This vertical force is called *buoyancy* and is equal in magnitude to the *weight of the displaced fluid*.”

Displaced mass (kg) = volume displaced (m<sup>3</sup>) x density of the fluid (kg m<sup>-3</sup>)

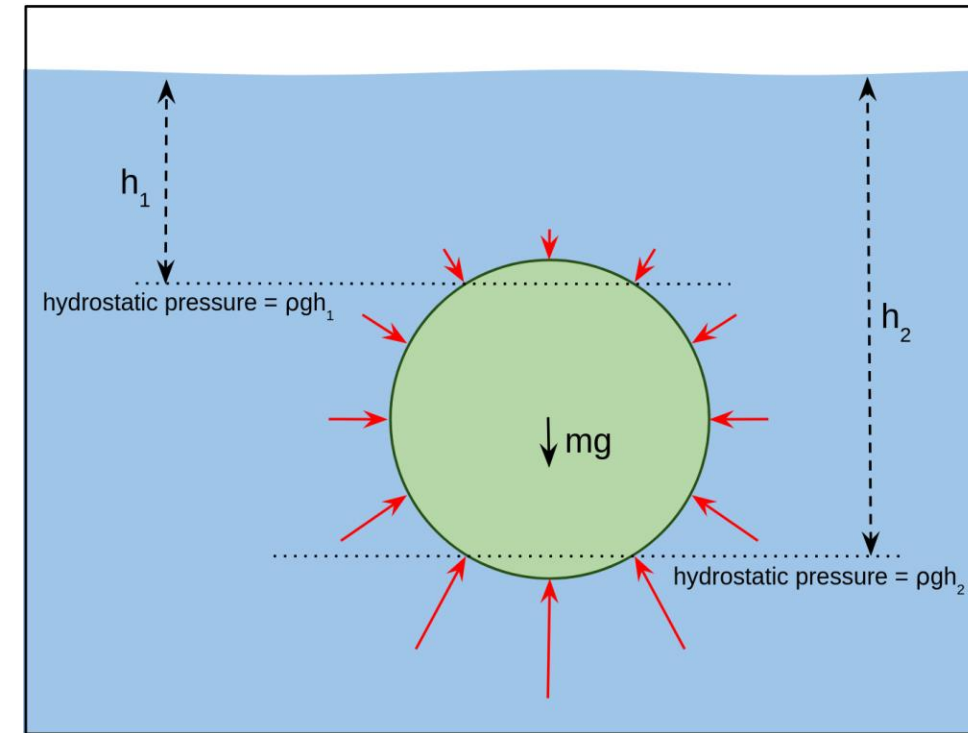
Buoyancy force (N) = volume displaced (m<sup>3</sup>) x density of the fluid (kg m<sup>-3</sup>) x gravitational acceleration (m s<sup>-2</sup>)  
=  $V\rho g$



# Hydrostatic pressure

The hydrostatic pressure is the pressure exerted by a fluid on an immersed object, caused due to the force of gravity.

hydrostatic pressure = fluid density x gravitational acceleration x water height  
 $= \rho g h$



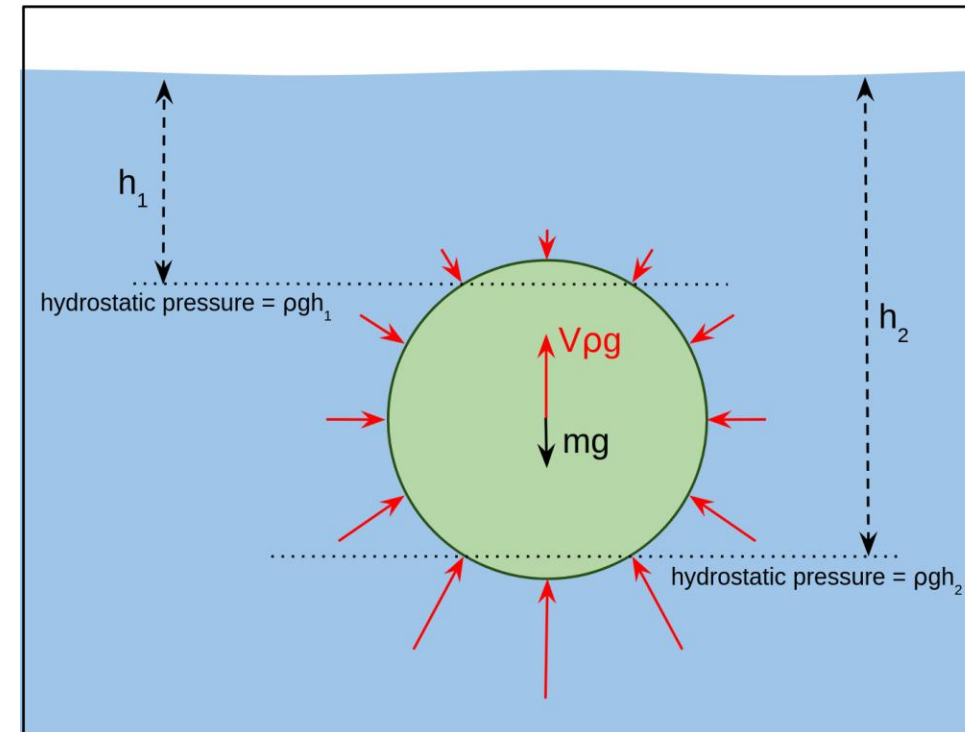
# Buoyancy force

**The hydrostatic pressure is the pressure exerted by a fluid on an immersed object, caused due to the force of gravity.**

hydrostatic pressure = fluid density x gravitational acceleration x water height  
 $= \rho g h$

## The buoyancy force

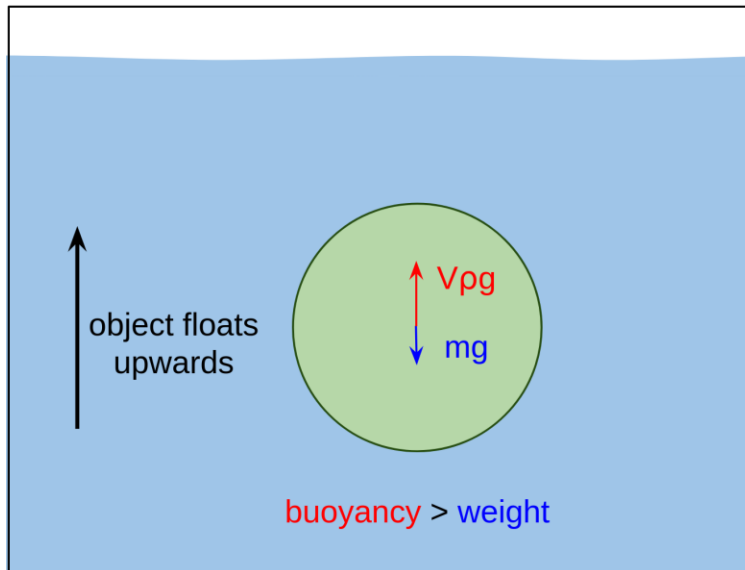
- The hydrostatic pressure acting on the body varies along the vertical axis, with the water height.
- The lateral components of the hydrostatic force cancel each other out, since they are equal and in opposite directions.
- Due to the hydrostatic pressure difference in the vertical axis, the vertical components of the hydrostatic force creates an upwards resultant force, i.e., the buoyancy force ( $V\rho g$ ).



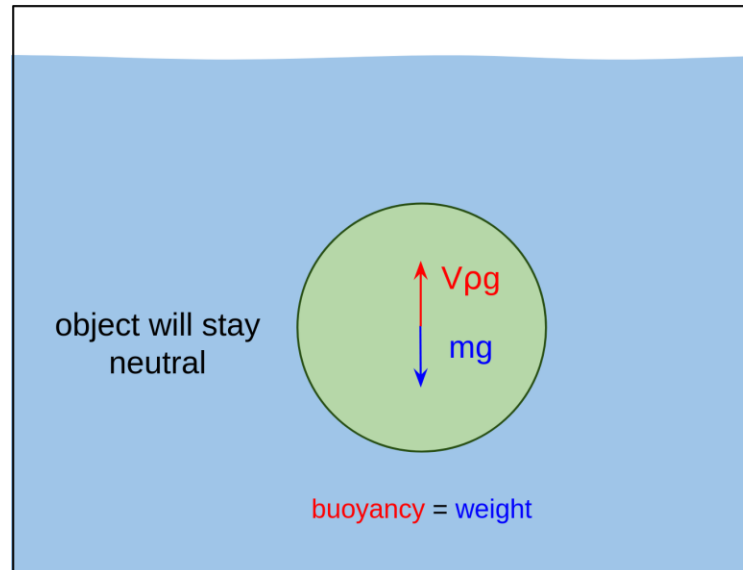
# Weight and buoyancy

- If the buoyancy is larger than the weight of the object, it will float upwards.
- If the buoyancy is equal to the weight, it will stay natural in the water column.
- If the buoyancy is smaller than the weight, it will sink to the bottom.

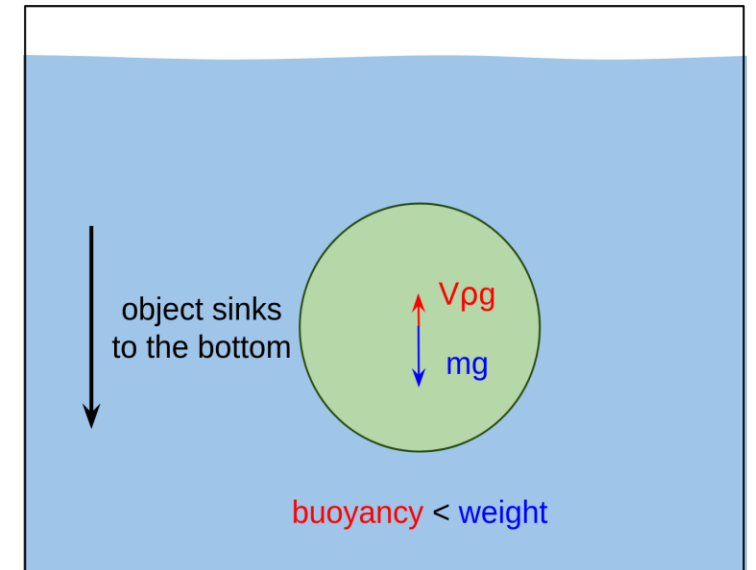
Positive buoyancy



Neutral buoyancy



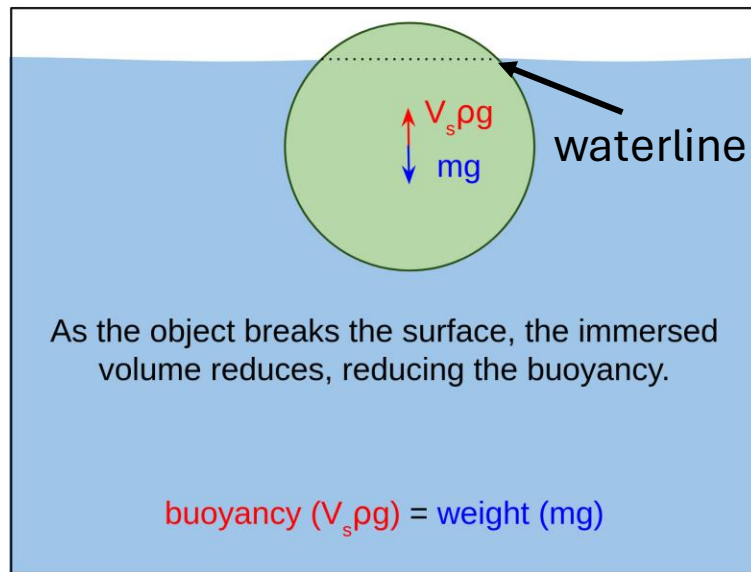
Negative buoyancy



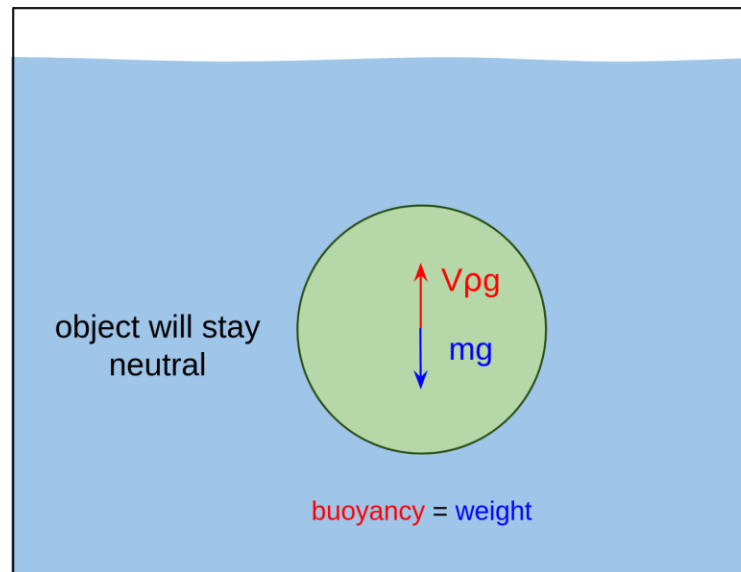
# Weight and buoyancy

- If the buoyancy is larger than the weight of the object, it will float upwards.
  - When at surface, object will stick out of the water until the immersed volume (i.e., the buoyancy) reduces until buoyancy is equal to the weight.
- If the buoyancy is equal to the weight, it will stay natural in the water column.
- If the buoyancy is smaller than the weight, it will sink to the bottom.

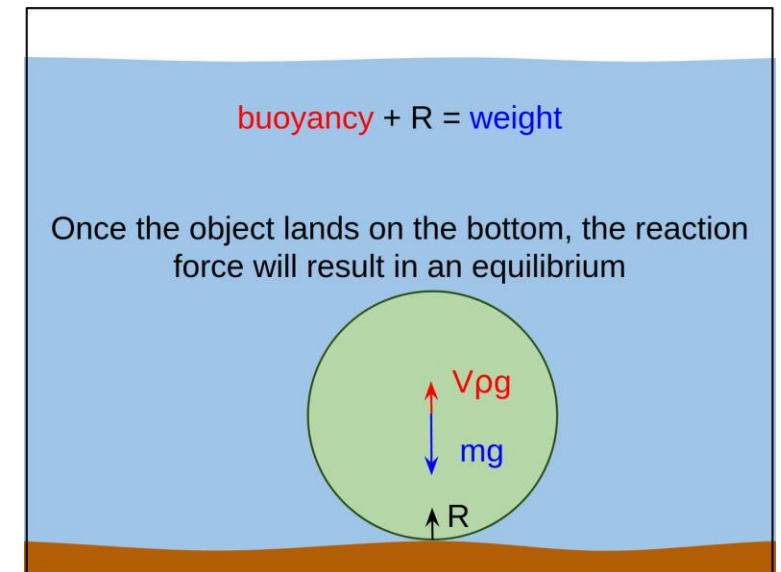
Positive buoyancy



Neutral buoyancy



Negative buoyancy



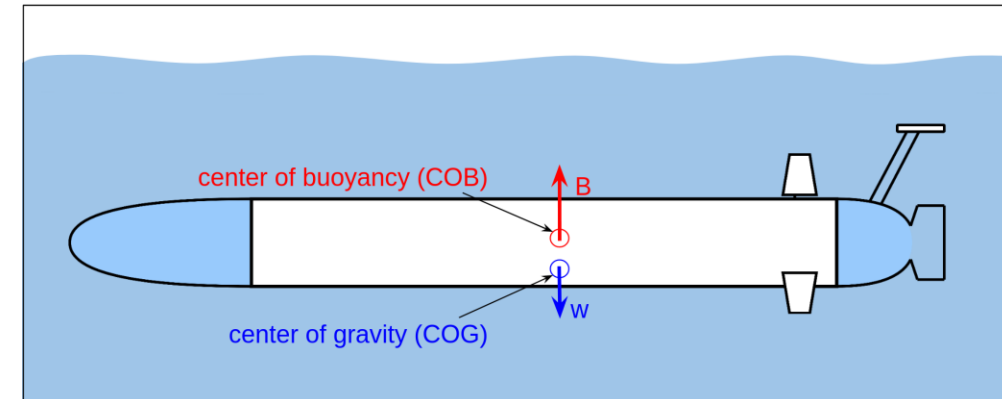
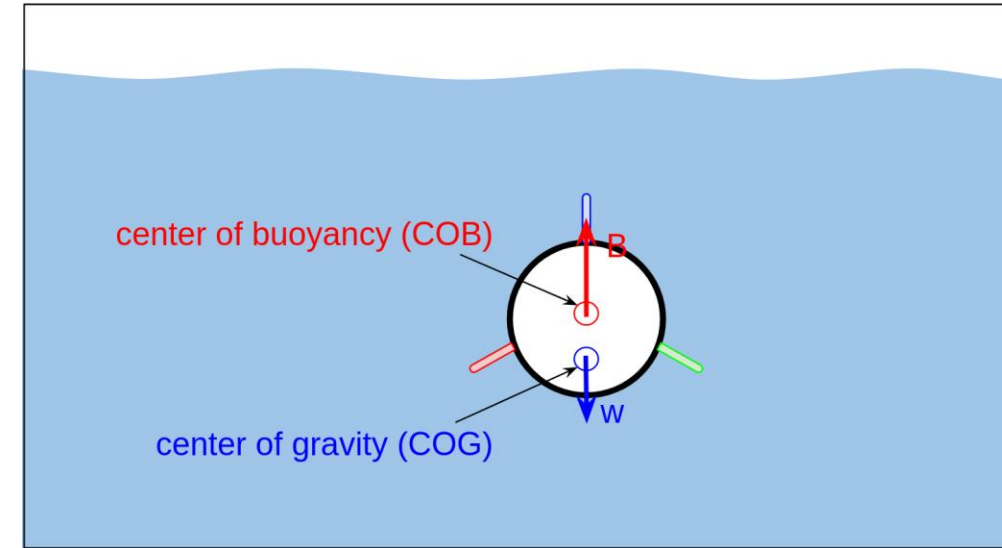
# Center of buoyancy and center of gravity

## Center of gravity ( $C_g$ ):

- The point where the gravitational force (i.e., the weight) acts upon.
- The position of center of gravity:
  - Longitudinal center of gravity (LCG) - longitudinal distance from origin to  $C_g$
  - Vertical center of gravity (VCG) - vertical distance from origin to  $C_g$
  - Transverse center of gravity (TCG) - transverse distance from origin to  $C_g$

## Center of buoyancy ( $C_b$ ):

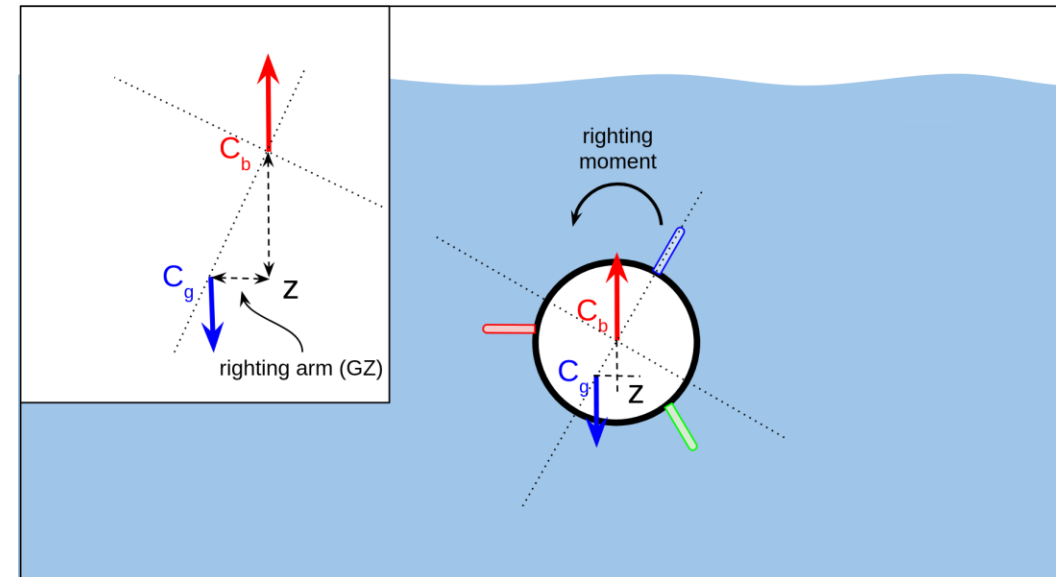
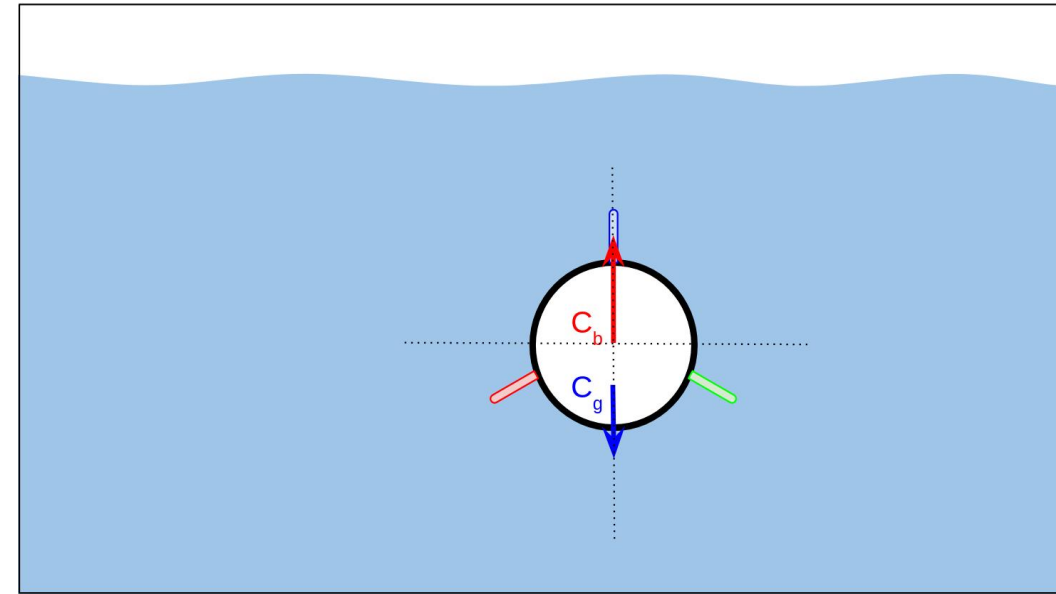
- The point where the buoyancy force acts upon. I.e., the center of gravity for the volume of water which a hull displaces.
- Varies with the shape of the non-free-flood hull.
- The position of center of buoyancy:
  - Longitudinal center of buoyancy (LCB) - longitudinal distance from origin to  $C_b$
  - Vertical center of buoyancy (VCB) - vertical distance from origin to  $C_b$
  - Transverse center of buoyancy (TCB) - transverse distance from origin to  $C_b$



# Submerged transverse stability of an AUV

## Positive stability:

- If the center of buoyancy is above the center of gravity (i.e., VCB is above VCG), the AUV is in positive stability condition.
- If the AUV inclined due to an external force, a righting moment will be created to return the vehicle to its original position.

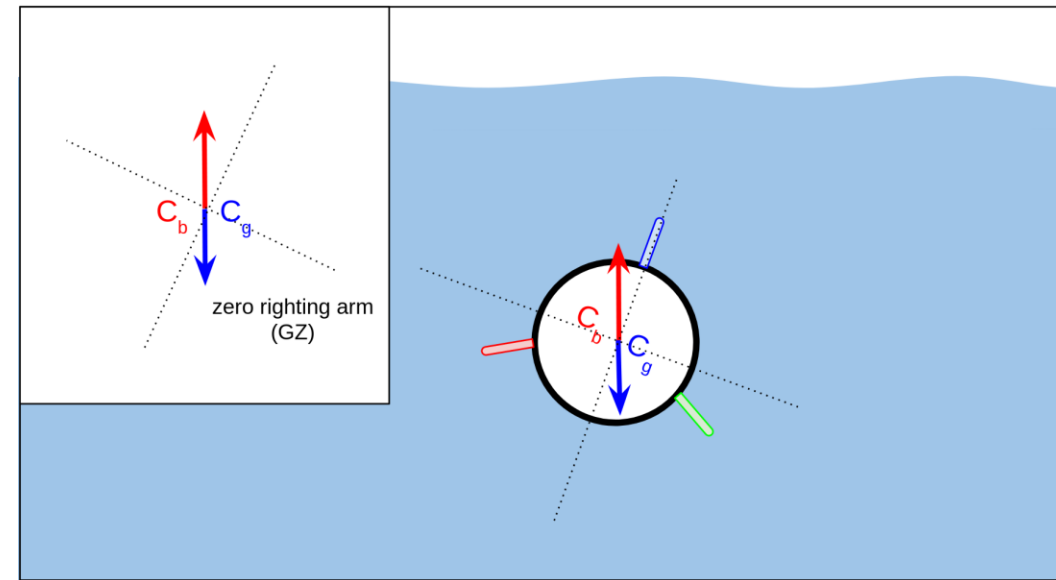
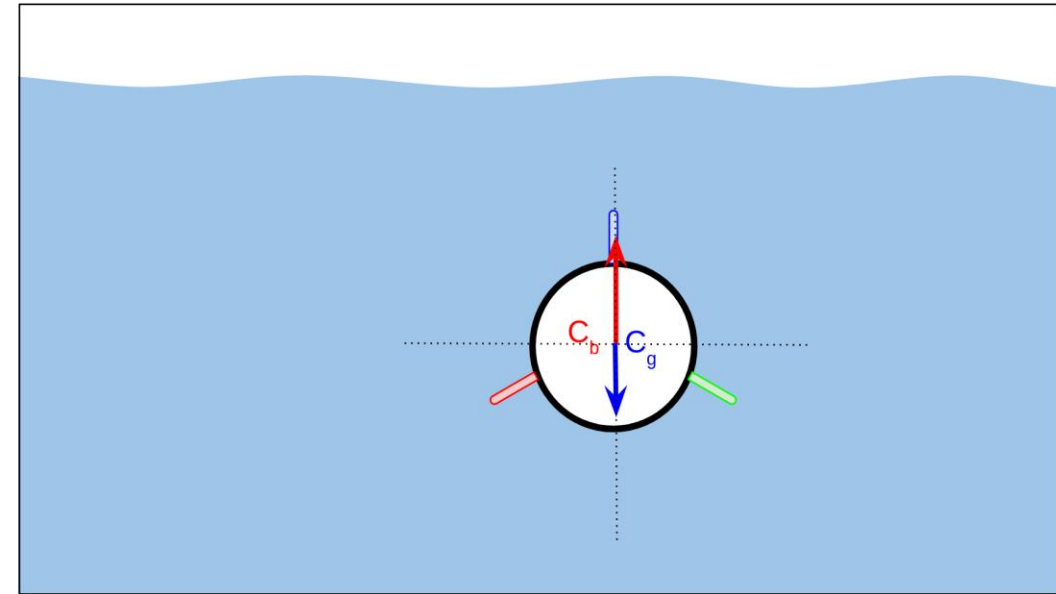




# Submerged transverse stability of an AUV

## Neutral stability:

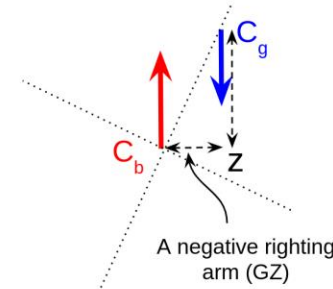
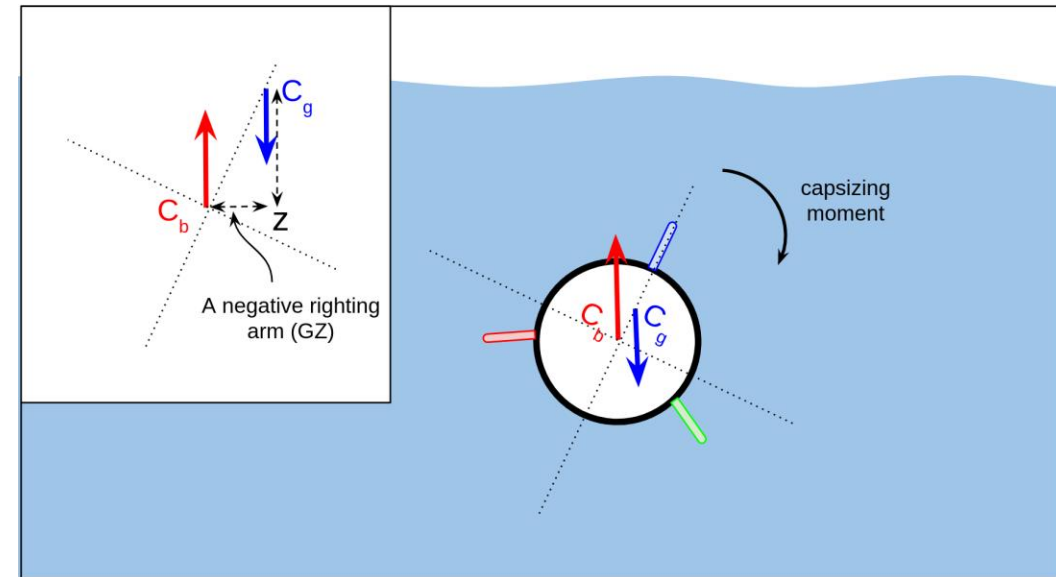
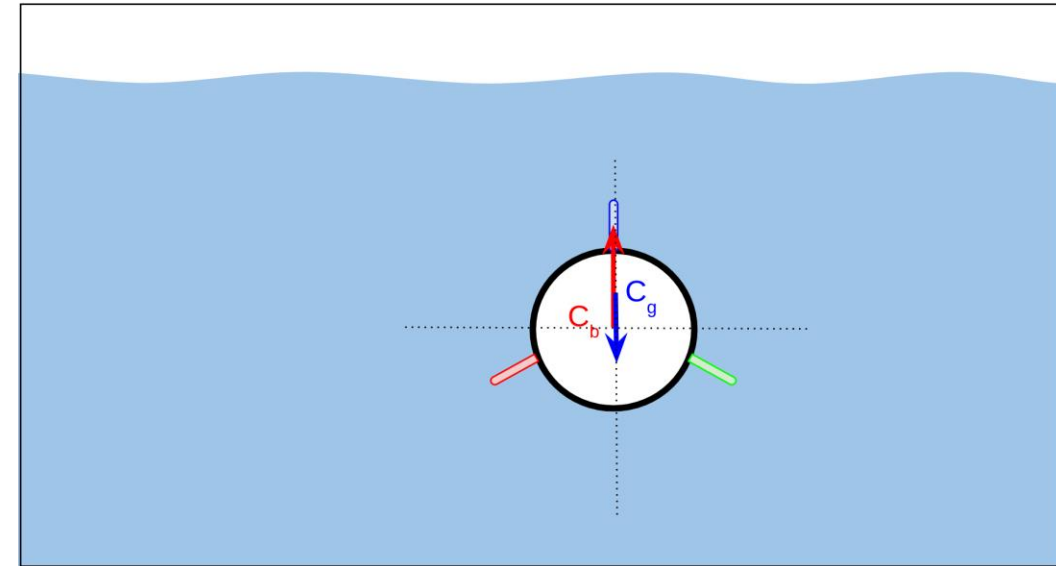
- If the center of buoyancy and the center of gravity are in the same vertical position (i.e., VCB and VCG are equal)
- If the AUV inclined due to an external force, no righting arm is created. It will stay in at the same roll angle.



# Submerged transverse stability of an AUV

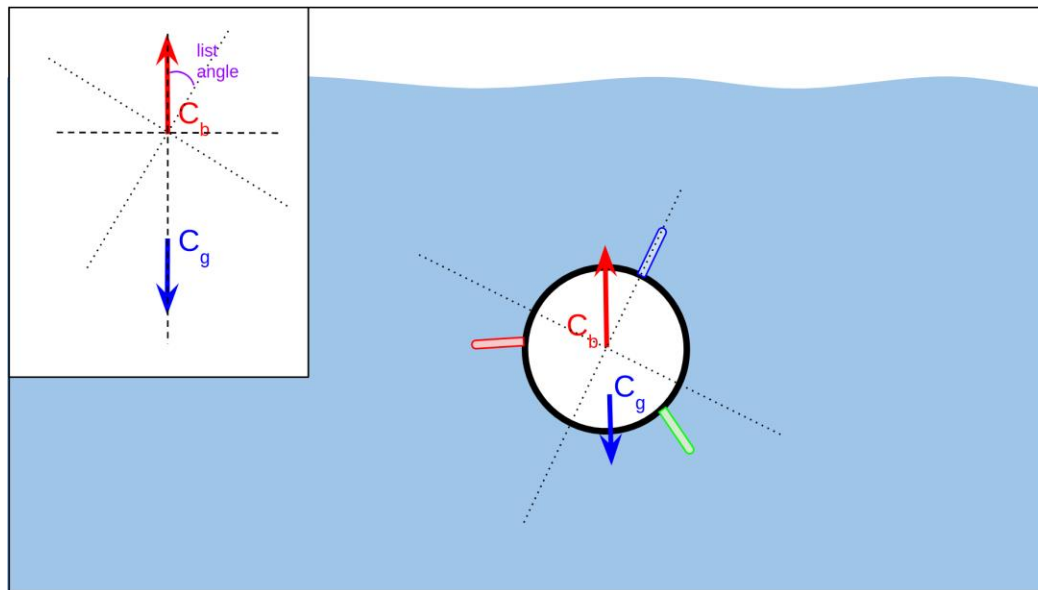
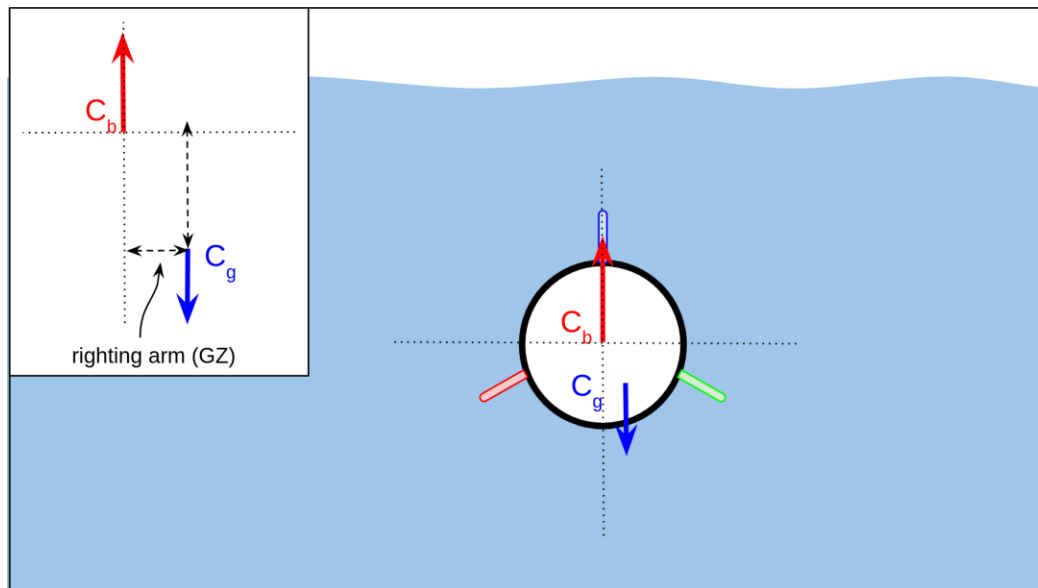
## Negative stability:

- If the center of buoyancy is below the center of gravity (i.e., VCB is below VCG), the vehicle is negatively stable.
- A small inclination can result in the AUV turning upside down due to the capsizing moment.



# List angle

- If the transverse center of gravity (TCG) is not equal to transverse center of buoyancy (TCB), a negative righting moment will be created.
- Righting moment will result in a permanent steady-state roll angle, called the list angle.
- Some AUVs are ballasted to have a list angle that counteracts the propeller torque.

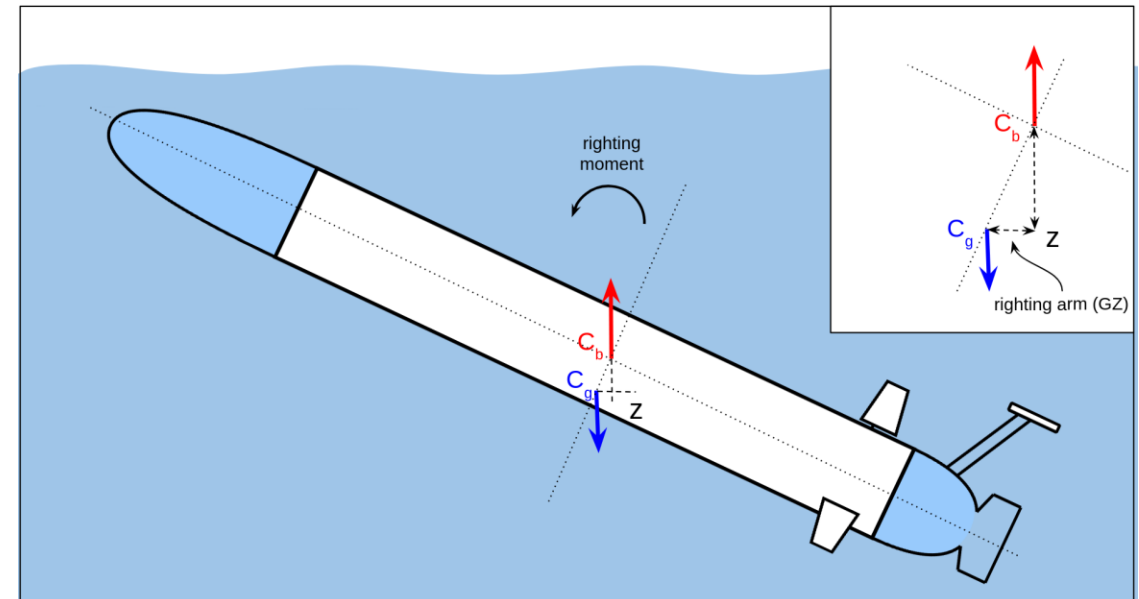
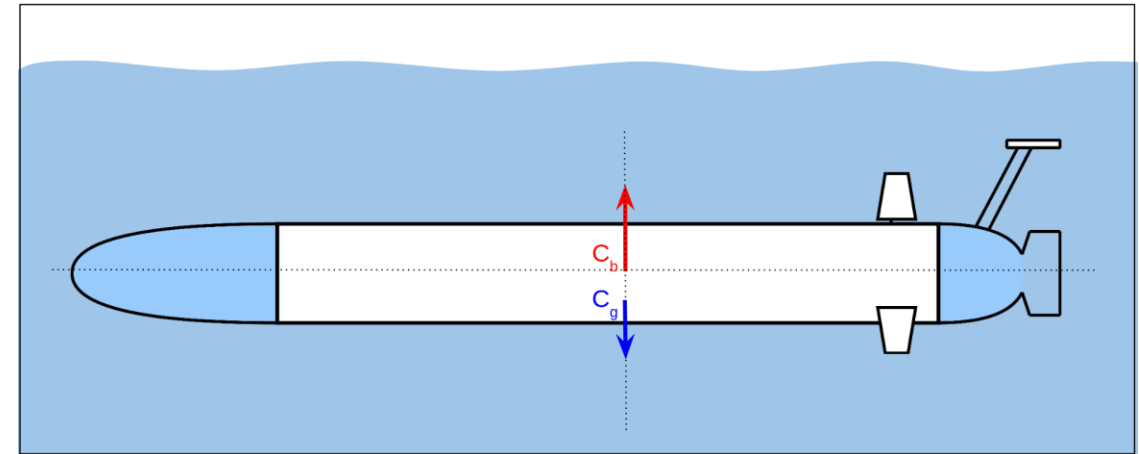


# Submerged longitudinal stability of an AUV

Similar to the transverse stability, the position of VCB and VCG will result in a positive, neutral or negative longitudinal stability.

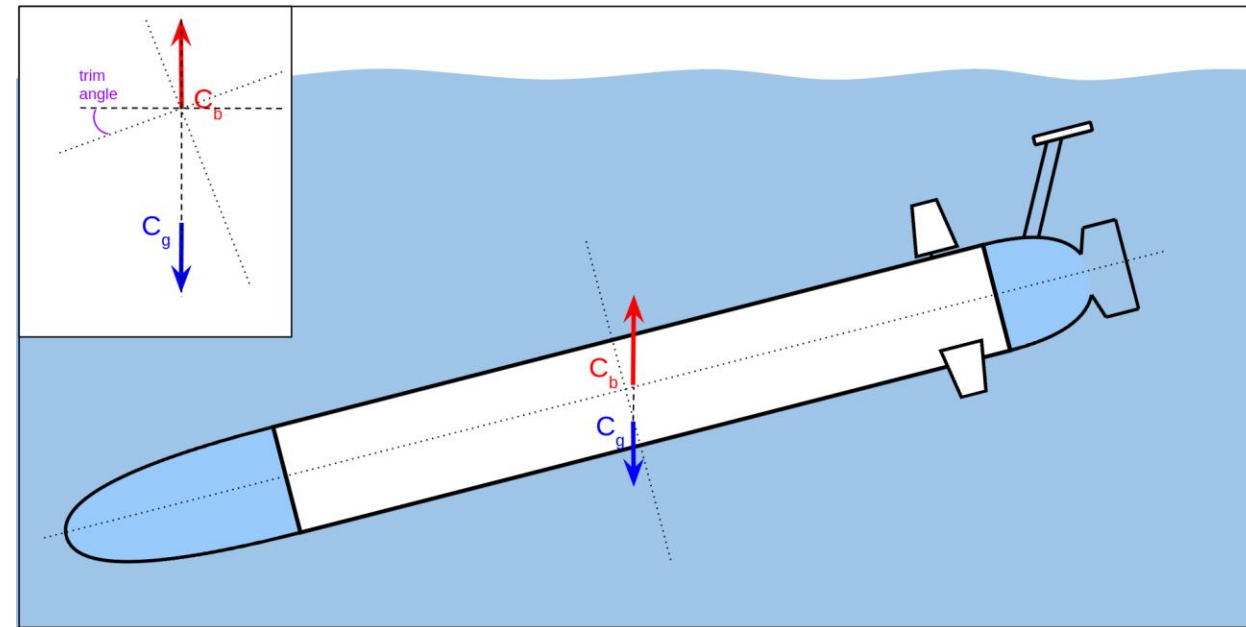
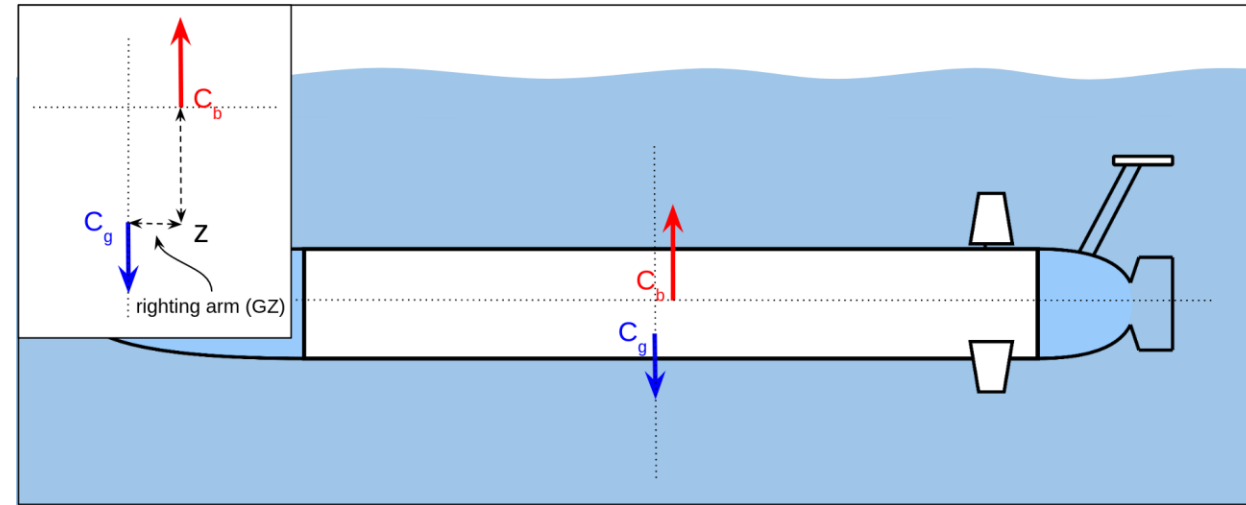
The figures show the positive stability condition:

- The VCB is above VCG
- If the AUV inclined due to an external force, a righting moment will be created to return the vehicle to its original position.



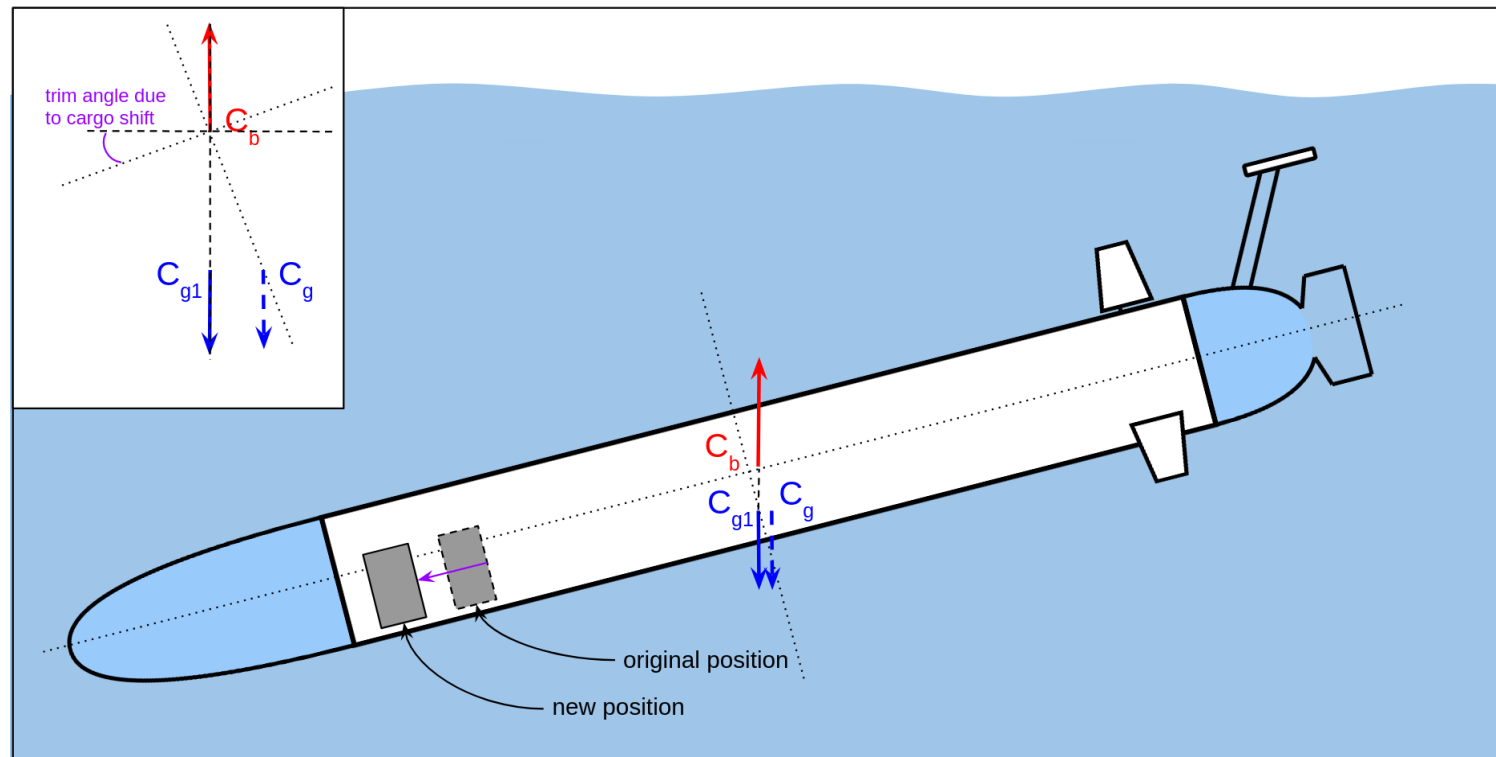
# Trim angle

- If the longitudinal center of gravity (LCG) is not equal to longitudinal center of buoyancy (LCB), a negative righting moment will be created.
- This righting moment will result in a permanent steady-state pitch angle, called the trim angle.
- Most torpedo-shaped, small AUVs need to be ballasted with a small trim angle (nose down), to support the initial dive.



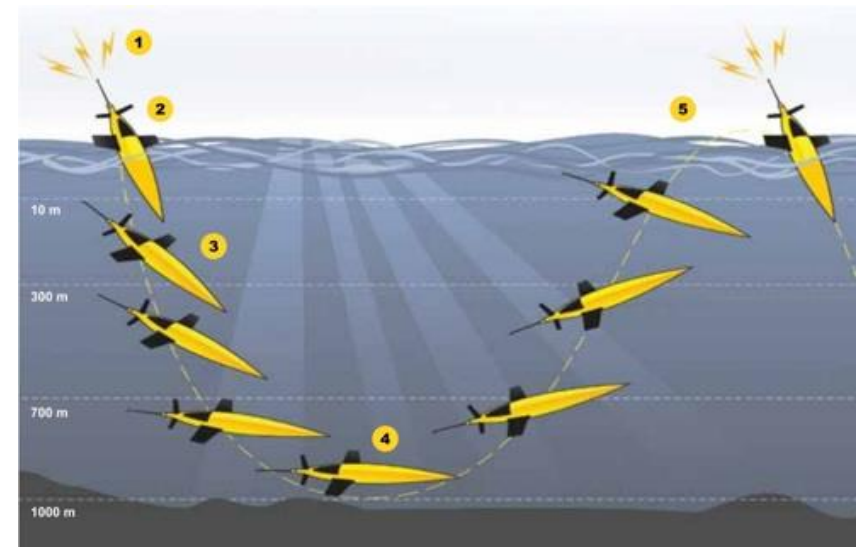
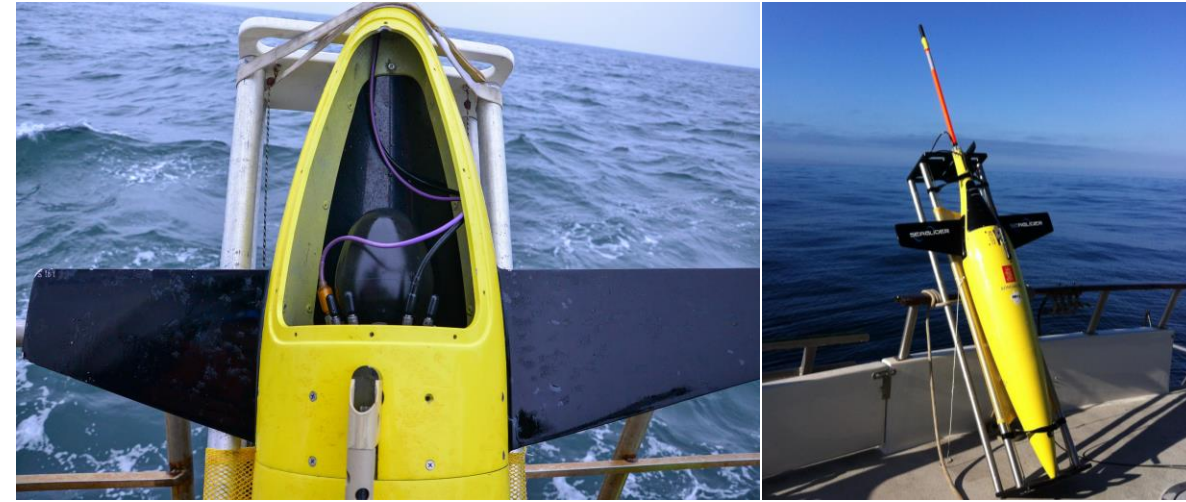
# Mid-mission cargo shifting

- If hardware equipment are not properly secured, they may shift due to the motion of the vehicle.
- Such weight shifts will result in unintended trim and list angles.
- This may result in mission failures or vehicle loss.



# Buoyancy engines and variable ballast tanks

- Buoyancy engines can actively control the buoyancy of the vehicle.
- Types of buoyancy engines include:
  - Inflating/deflating oil bladders.
  - Pump-based variable ballast tanks.





# Buoyancy engines and variable ballast tanks

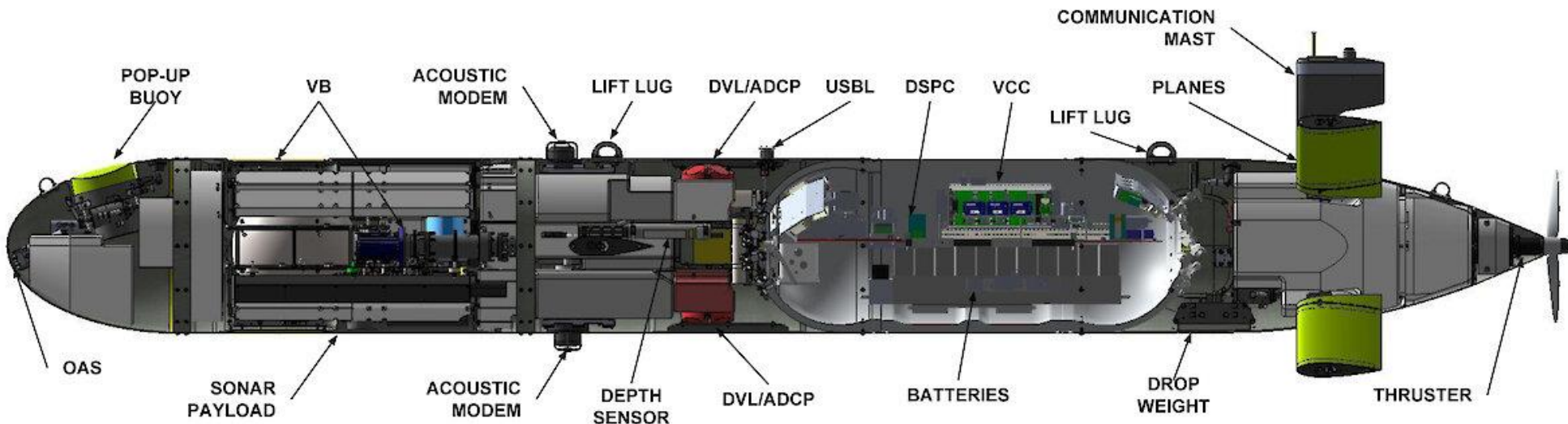
- Buoyancy engines can actively control the buoyancy of the vehicle.
- Types of buoyancy engines include:
  - Inflating/deflating oil bladders.
  - Pump-based variable ballast tanks.
- Buoyancy engines can be used to:
  - Use gliding as a propulsion method
  - Land UUVs on the seabed
  - Park underneath ice-shelfs
  - Stay in the mid-water column without propulsion





# Drop-weights

- Drop-weights are primarily a safety feature.
- In emergency situations, the drop-weight can be released to make the vehicle positively buoyant.
- The drop-weight release can be triggered acoustically by a human operator, or by the vehicle's autonomy system.



End