



MIT 2.S01 Introduction to
Autonomous Underwater
Vehicles

Lecture 8: Hydrostatics

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2026 Spring

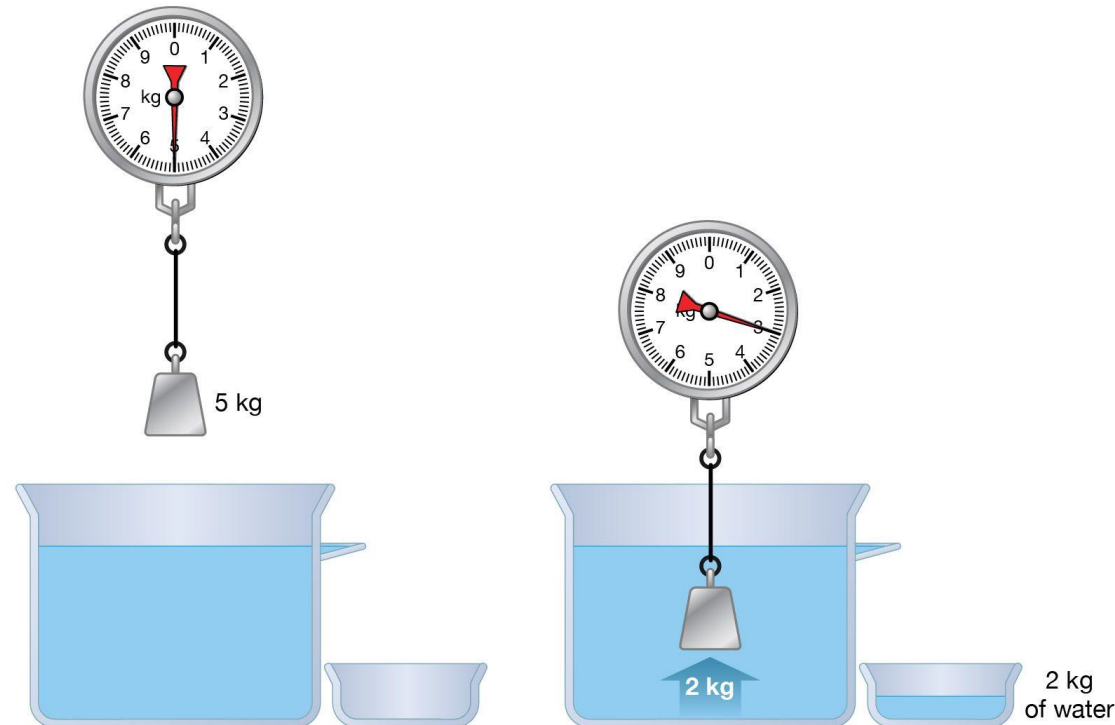


Archimedes' Principle

“An arbitrary shaped body immersed, either partly or fully, in a fluid will experience the effect of a net positive vertical force originating from the fluid pressure. This vertical force is called **buoyancy** and **is equal in magnitude to the weight of the displaced fluid.**”

Displaced mass (kg) = volume displaced (m^3) x density of the fluid (kg m^{-3})

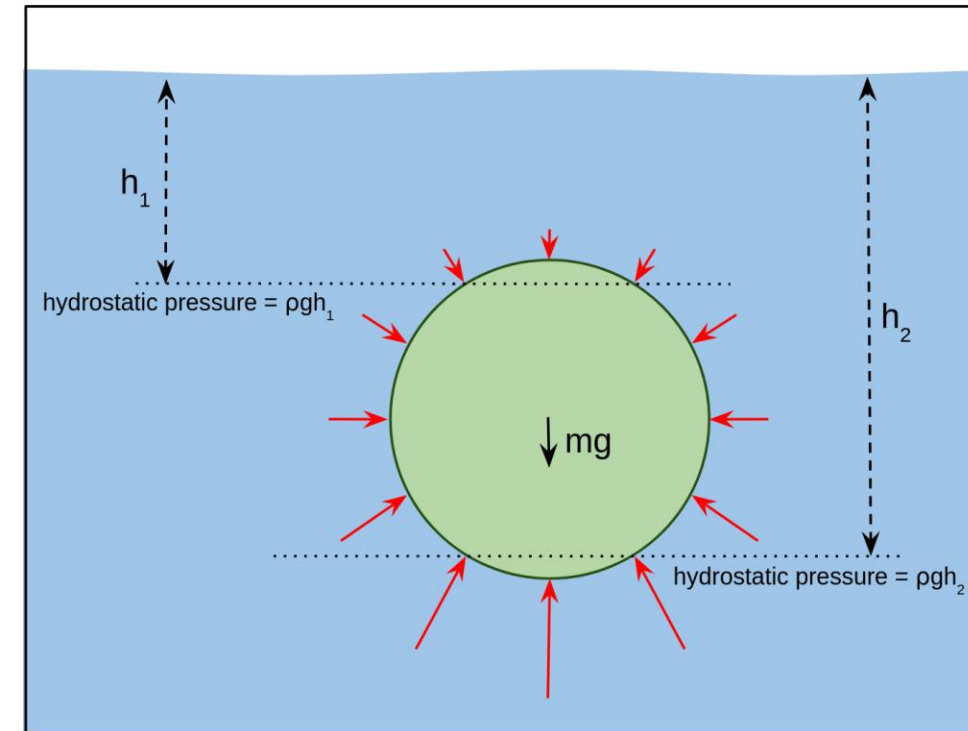
Buoyancy force (N) = volume displaced (m^3) x density of the fluid (kg m^{-3}) x gravitational acceleration (m s^{-2})
= $V \rho g$



Hydrostatic pressure

The hydrostatic pressure is the pressure exerted by a fluid on an immersed object, caused due to the force of gravity.

hydrostatic pressure = fluid density x gravitational acceleration x water height
 $= \rho g h$



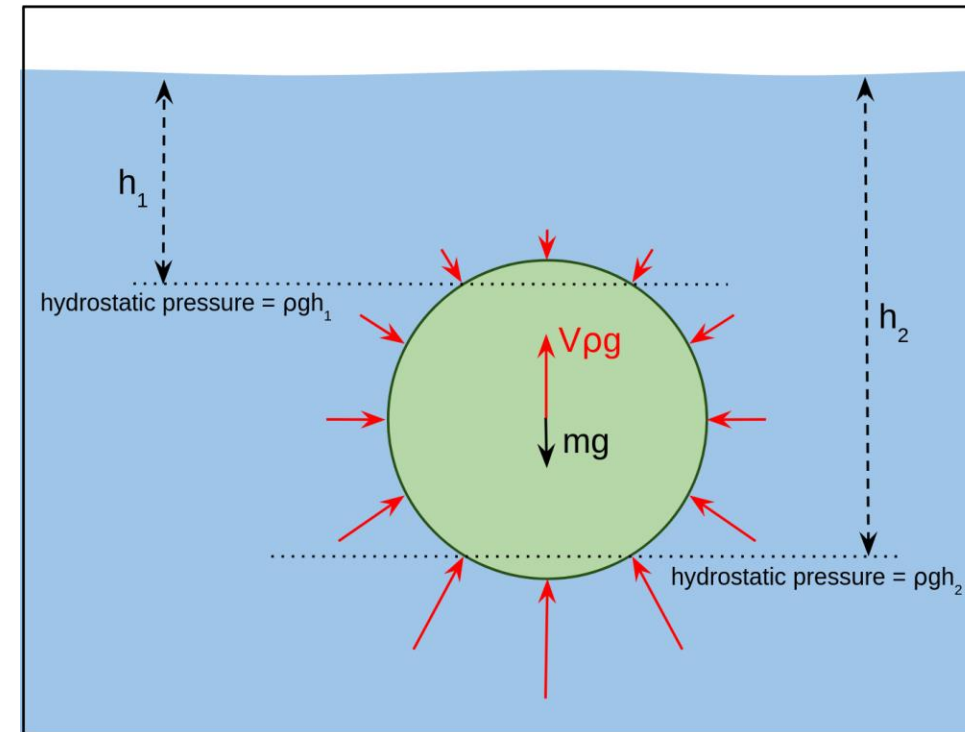
Buoyancy force

The hydrostatic pressure is the pressure exerted by a fluid on an immersed object, caused due to the force of gravity.

$$\begin{aligned}\text{hydrostatic pressure} &= \text{fluid density} \times \text{gravitational acceleration} \times \text{water height} \\ &= \rho g h\end{aligned}$$

The buoyancy force

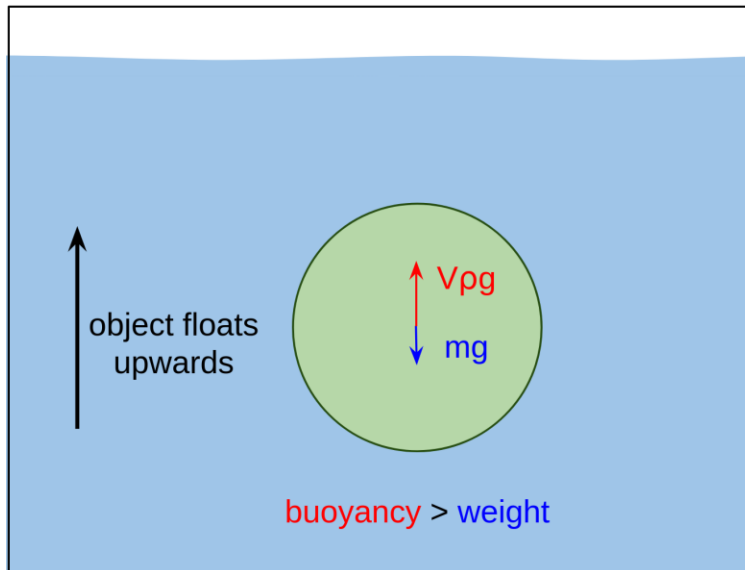
- The hydrostatic pressure acting on the body varies along the vertical axis, with the water height.
- The lateral components of the hydrostatic force cancel each other out, since they are equal and in opposite directions.
- Due to the hydrostatic pressure difference in the vertical axis, the vertical components of the hydrostatic force creates an upwards resultant force, i.e., the buoyancy force ($V\rho g$).



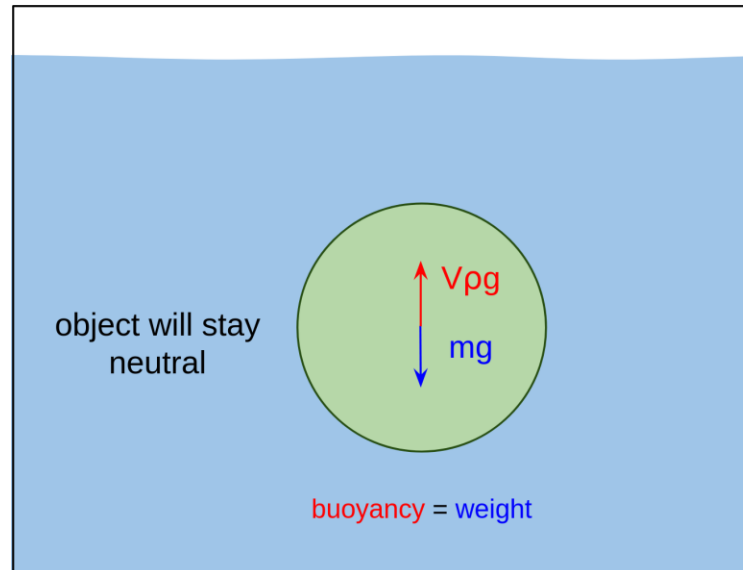
Weight and buoyancy

- If the buoyancy is larger than the weight of the object, it will float upwards.
- If the buoyancy is equal to the weight, it will stay natural in the water column.
- If the buoyancy is smaller than the weight, it will sink to the bottom.

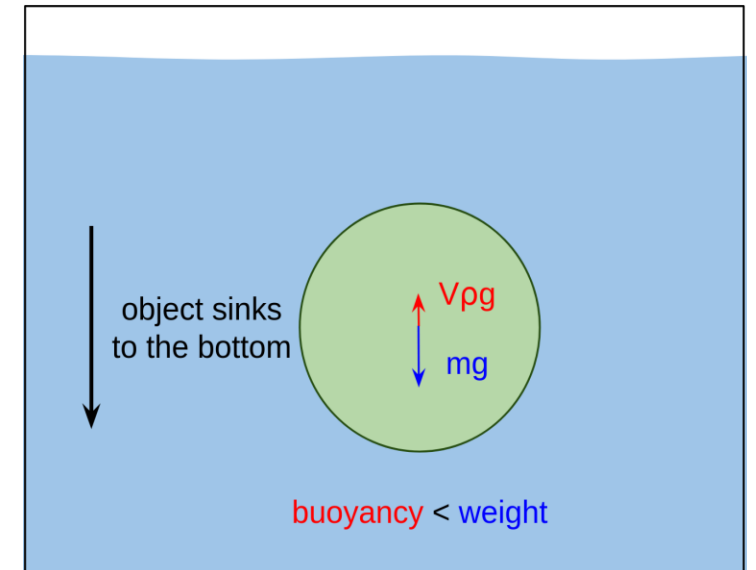
Positive buoyancy



Neutral buoyancy



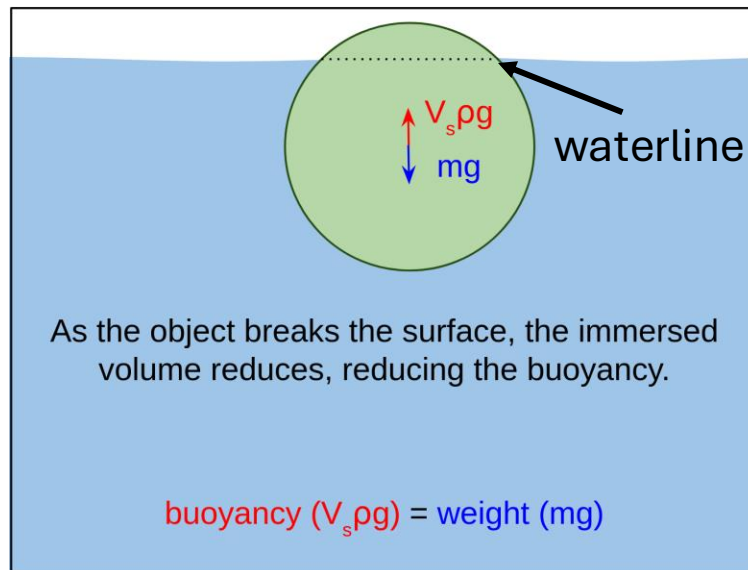
Negative buoyancy



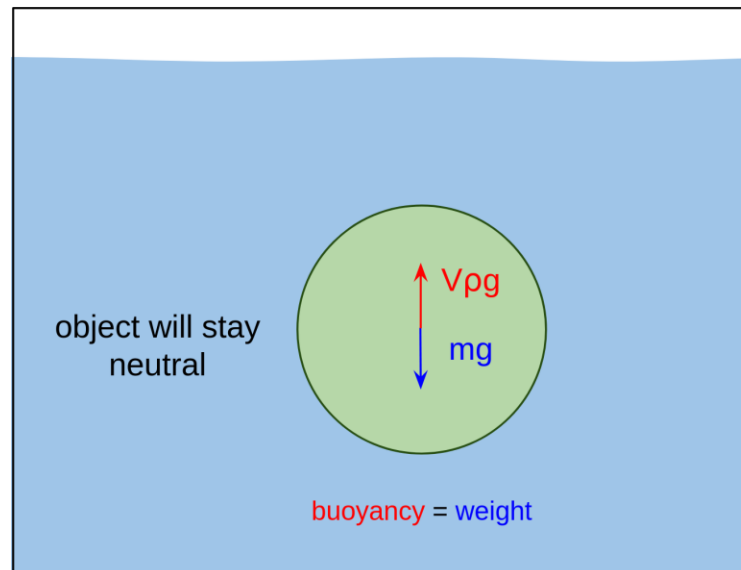
Weight and buoyancy

- If the buoyancy is larger than the weight of the object, it will float upwards.
 - When at the surface, the object will protrude above waterline until immersed volume decreases enough for the buoyant force to equal the object's weight.
- If the buoyancy is equal to the weight, it will stay neutral in the water column.
- If the buoyancy is smaller than the weight, it will sink to the bottom.

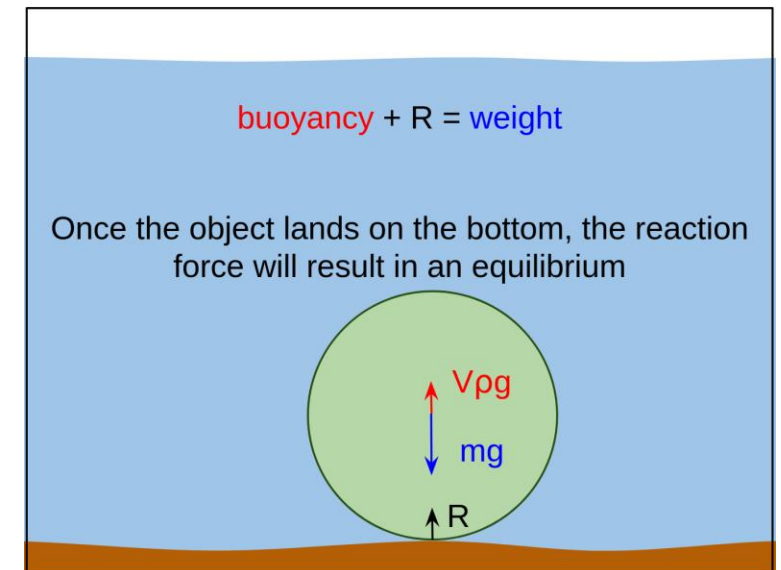
Positive buoyancy



Neutral buoyancy



Negative buoyancy



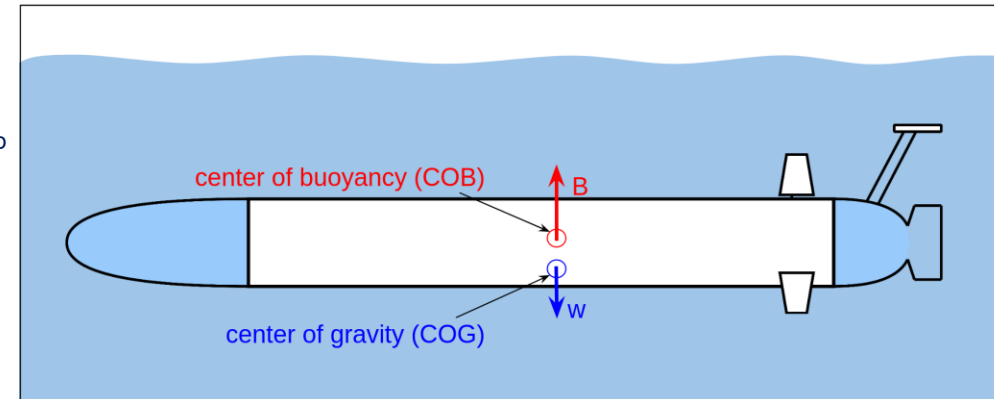
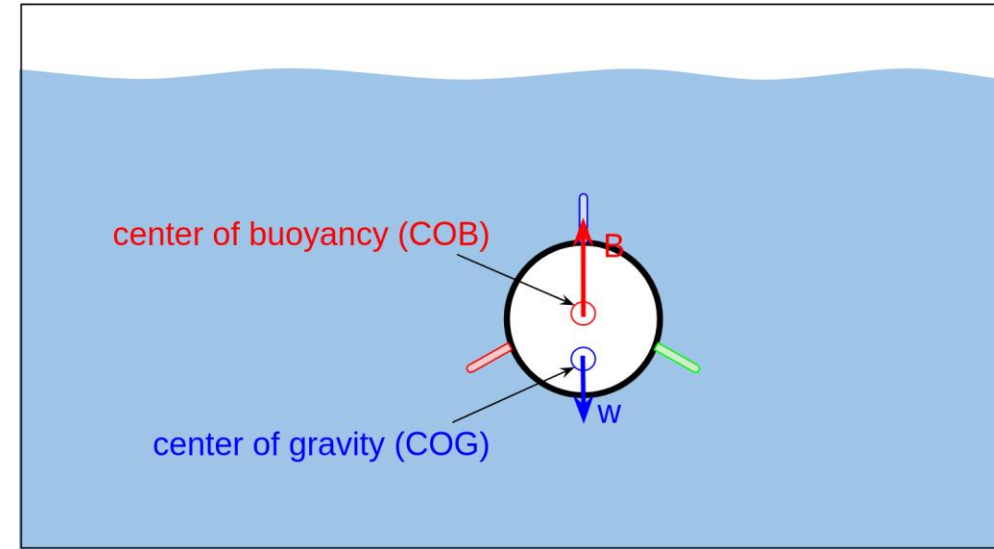
Center of buoyancy and center of gravity

Center of gravity (C_g):

- The point where the gravitational force (i.e., the weight) acts upon.
- The position of center of gravity:
 - Longitudinal center of gravity (LCG) - longitudinal distance from origin to C_g
 - Vertical center of gravity (VCG) - vertical distance from origin to C_g
 - Transverse center of gravity (TCG) - transverse distance from origin to C_g

Center of buoyancy (C_b):

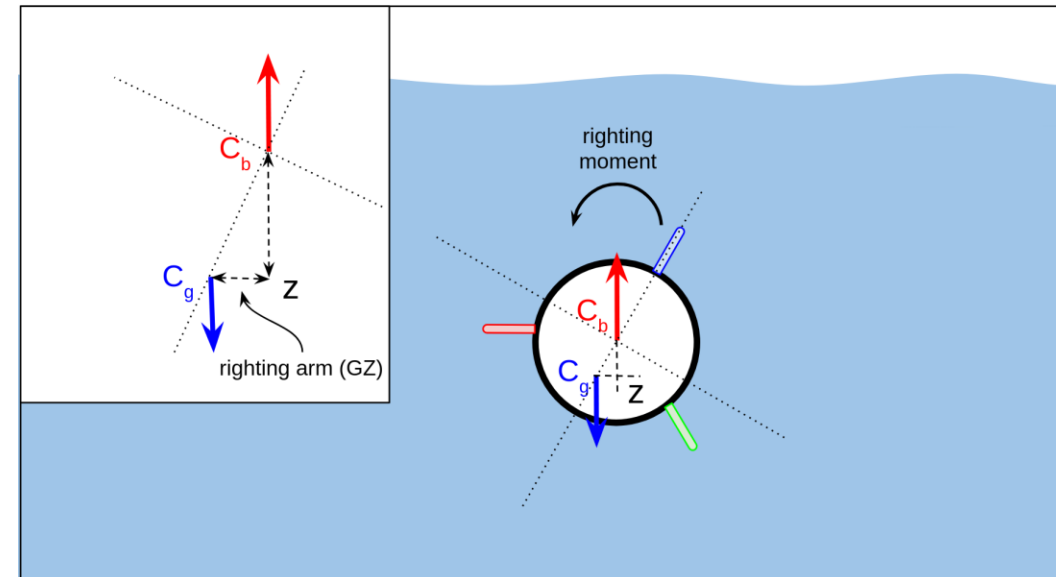
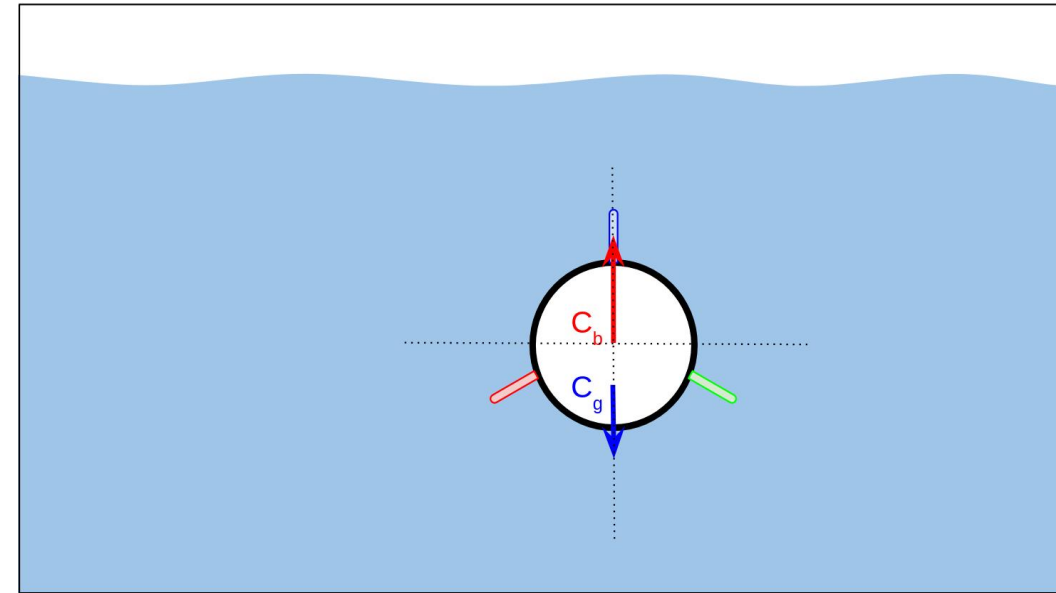
- The point where the buoyancy force acts upon. I.e., the center of gravity for the volume of water, which the hull displaces.
- Varies with the shape of the non-free-flood hull.
- The position of center of buoyancy:
 - Longitudinal center of buoyancy (LCB) - longitudinal distance from origin to C_b
 - Vertical center of buoyancy (VCB) - vertical distance from origin to C_b
 - Transverse center of buoyancy (TCB) - transverse distance from origin to C_b



Submerged transverse stability of an AUV

Positive stability:

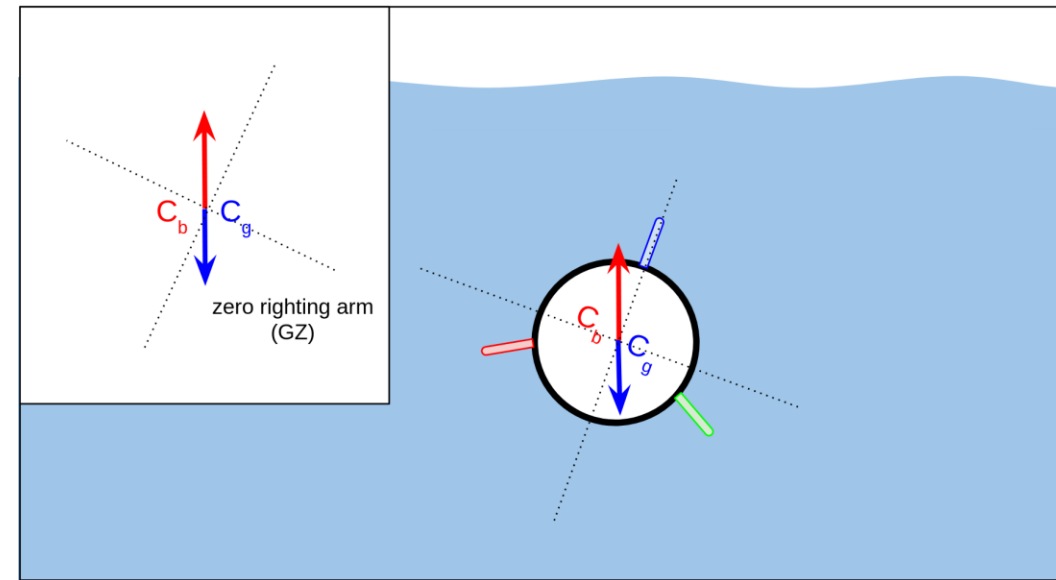
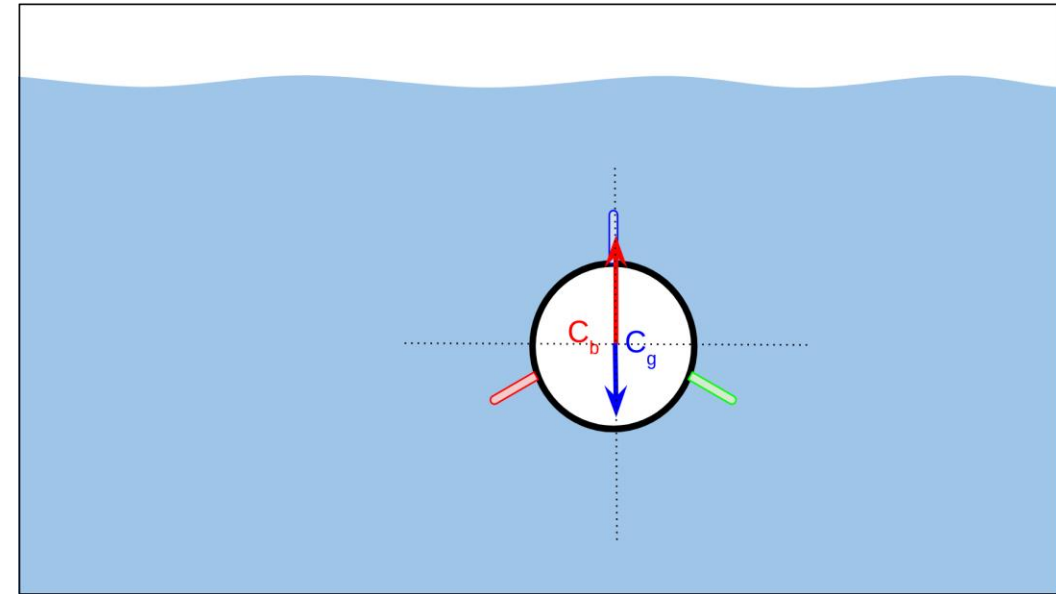
- If the center of buoyancy is above the center of gravity (i.e., VCB is above VCG), the AUV is in positive stability condition.
- If the AUV inclined due to an external force, a righting moment will be created to return the vehicle to its original position.



Submerged transverse stability of an AUV

Neutral stability:

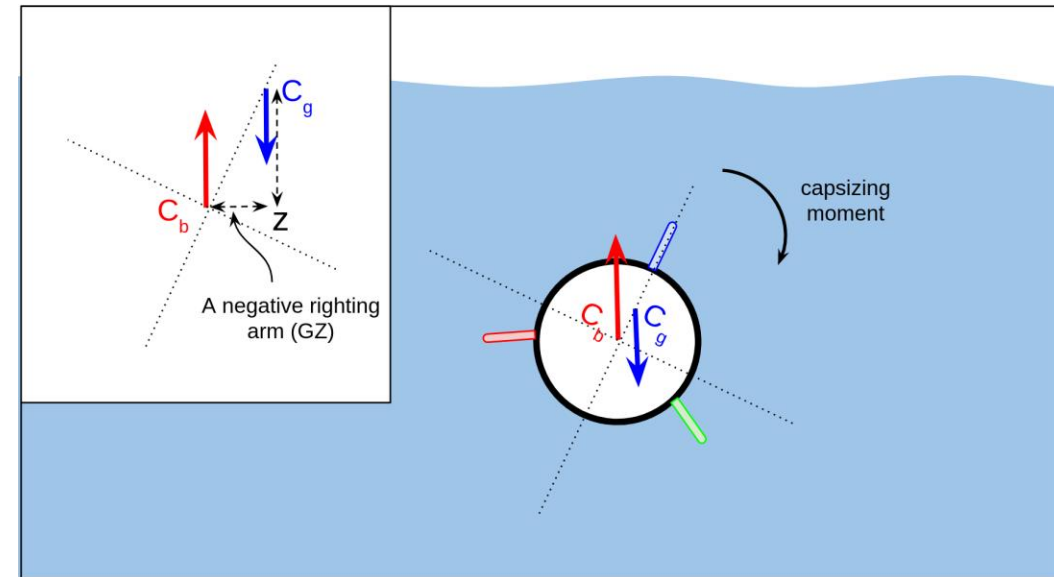
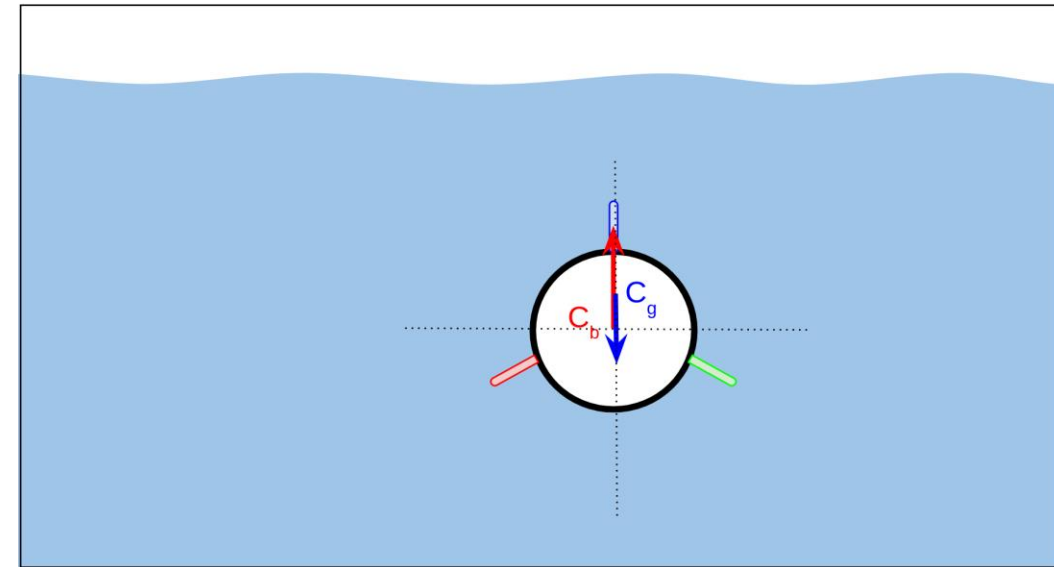
- If the center of buoyancy and the center of gravity are in the same vertical position (i.e., VCB and VCG are equal)
- If the AUV inclined due to an external force, no righting arm is created. It will stay in at the same roll angle.



Submerged transverse stability of an AUV

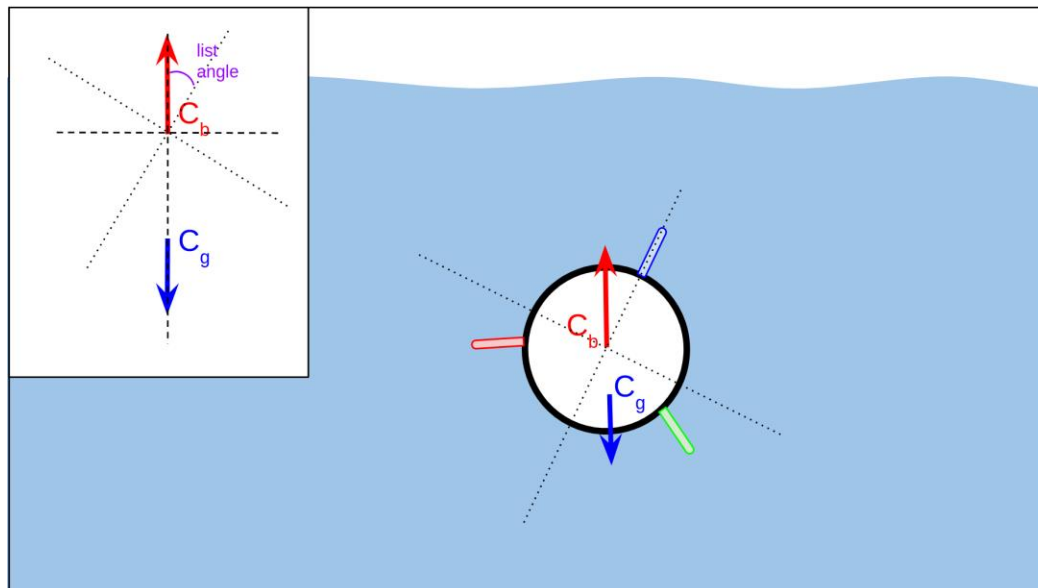
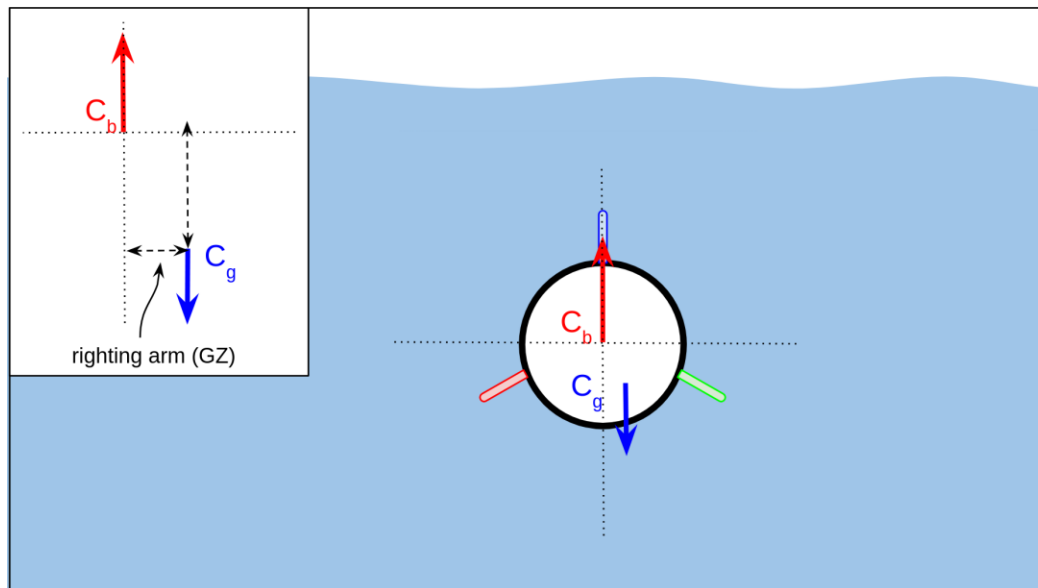
Negative stability:

- If the center of buoyancy is below the center of gravity (i.e., VCB is below VCG), the vehicle is negatively stable.
- A small inclination can result in the AUV turning upside down due to the capsizing moment.



List angle

- If the transverse center of gravity (TCG) is not equal to transverse center of buoyancy (TCB), a negative righting moment will be created.
- Righting moment will result in a permanent steady-state roll angle, called the list angle.
- Some AUVs are ballasted to have a list angle that counters the propeller torque.

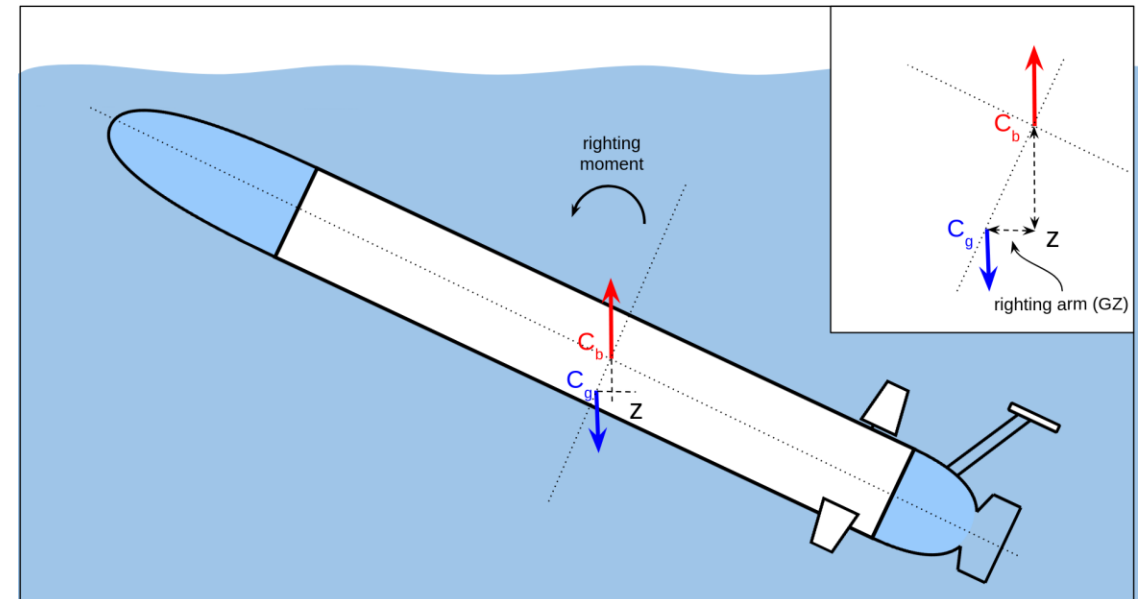
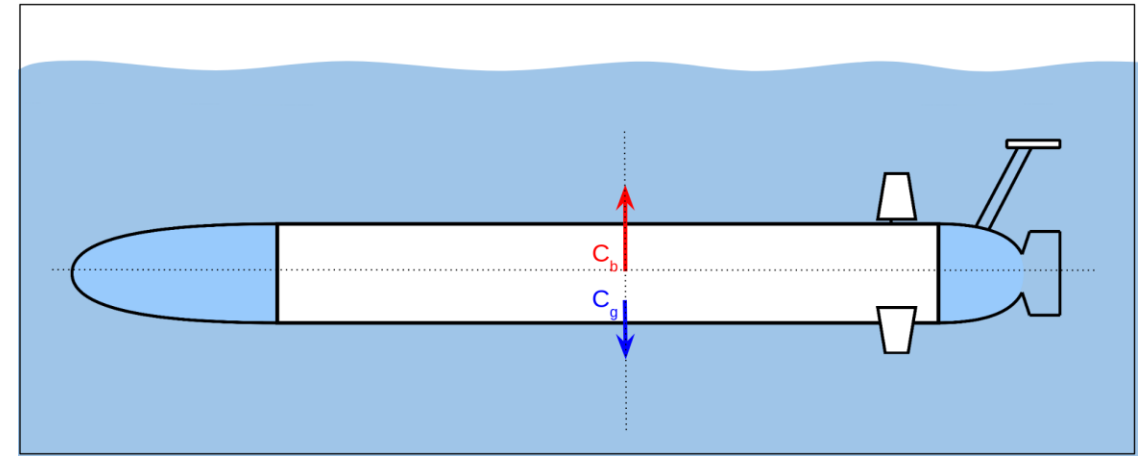


Submerged longitudinal stability of an AUV

Like the transverse stability, the position of VCB and VCG will result in a positive, neutral or negative longitudinal stability.

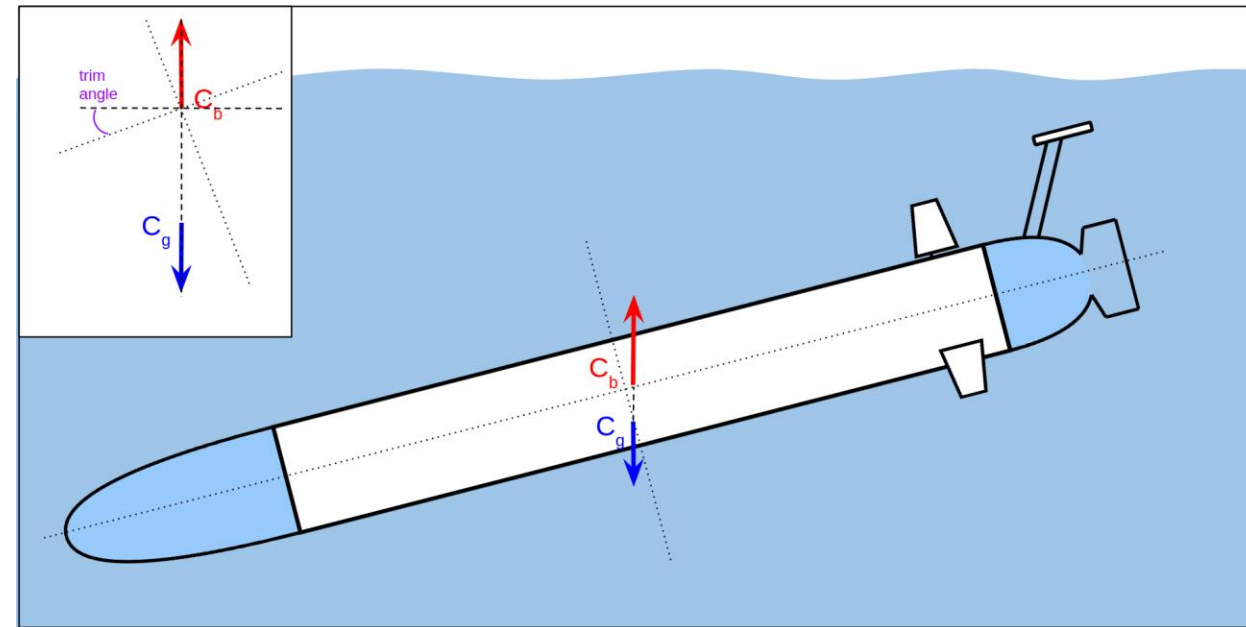
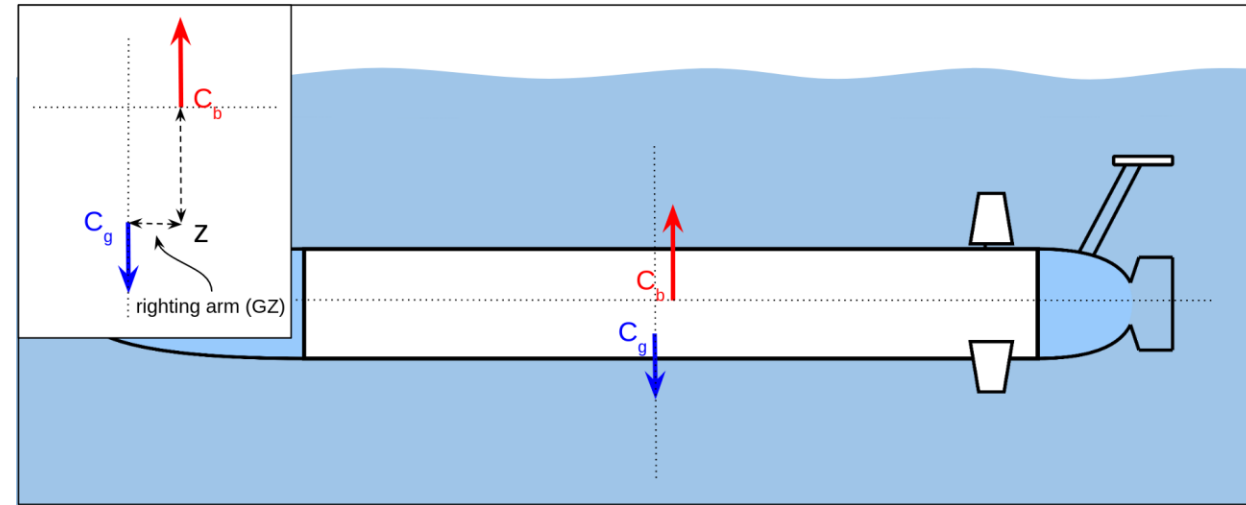
The figures show the positive stability condition:

- The VCB is above VCG
- If the AUV inclined due to an external force, a righting moment will be created to return the vehicle to its original position.



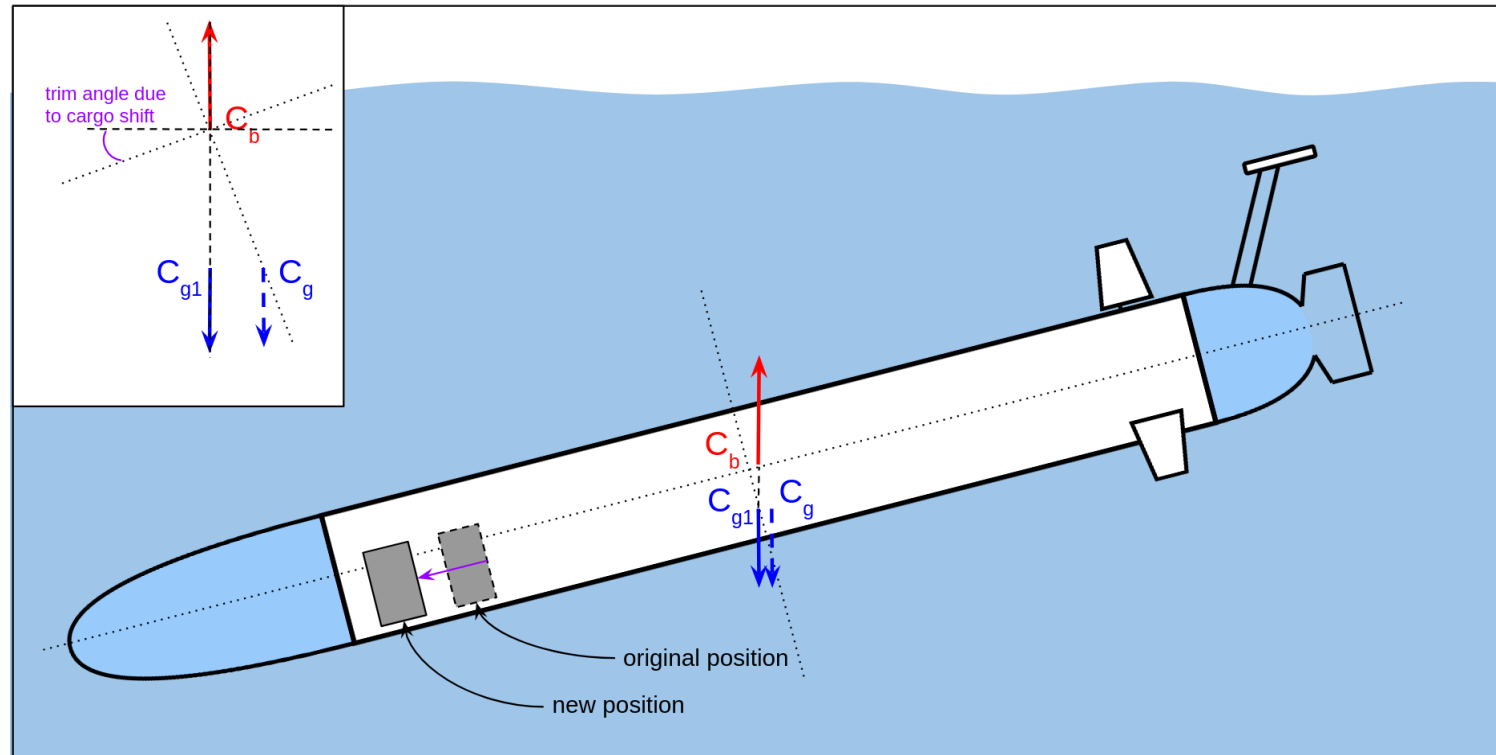
Trim angle

- If the longitudinal center of gravity (LCG) is not equal to longitudinal center of buoyancy (LCB), a negative righting moment will be created.
- This righting moment will result in a permanent steady-state pitch angle, called the trim angle.
- Most torpedo-shaped, small AUVs need to be ballasted with a small trim angle (nose down), to support the initial dive.



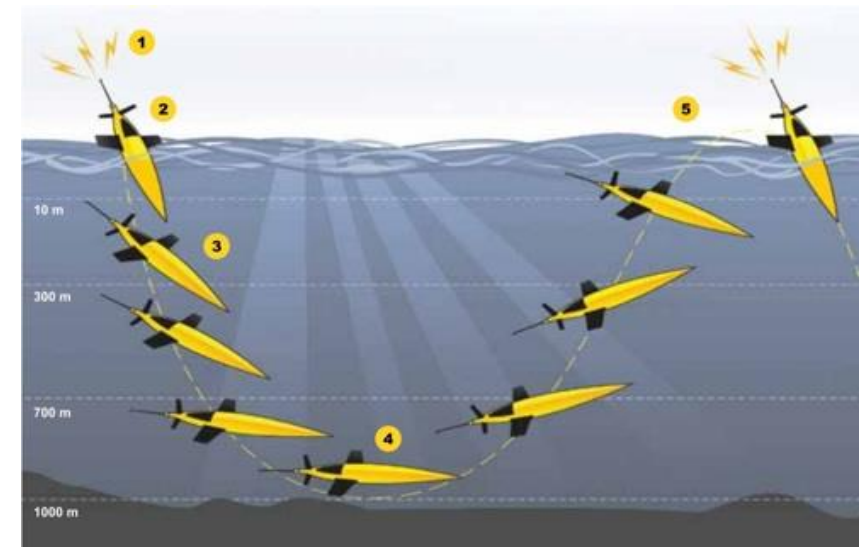
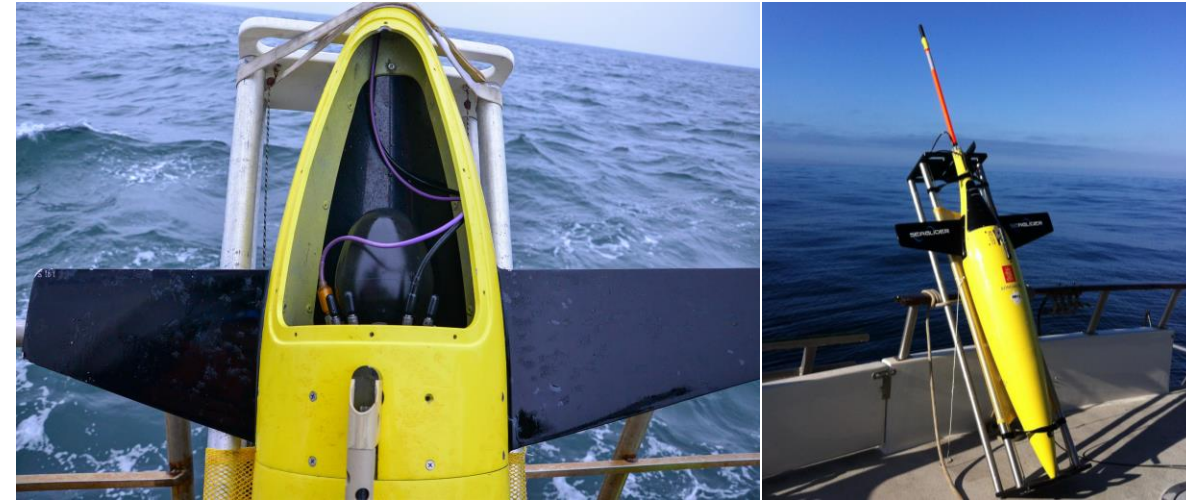
Mid-mission cargo shifting

- If hardware equipment are not properly secured, they may shift due to the motion of the vehicle.
- Such weight shifts will result in unintended trim and list angles.
- This may result in mission failures or vehicle loss.



Buoyancy engines and variable ballast tanks

- Buoyancy engines can actively control the buoyancy of the vehicle.
- Types of buoyancy engines include:
 - Inflating/deflating oil bladders.
 - Pump-based variable ballast tanks.



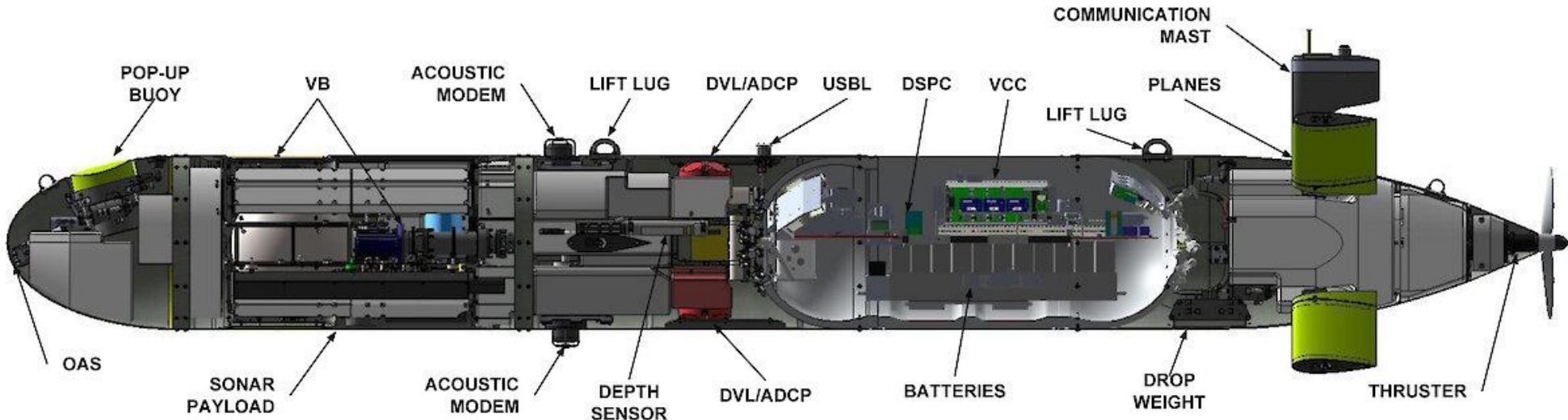
Buoyancy engines and variable ballast tanks

- Buoyancy engines can actively control the buoyancy of the vehicle.
- Types of buoyancy engines include:
 - Inflating/deflating oil bladders.
 - Pump-based variable ballast tanks.
- Buoyancy engines can be used to:
 - Use gliding as a propulsion method
 - Land UUVs on the seabed
 - Park underneath ice-shelfs
 - Stay in the mid-water column without propulsion



Drop-weights

- Drop-weights are primarily a safety feature.
- In emergency situations, the drop-weight can be released to make the vehicle positively buoyant.
- The drop-weight release can be triggered acoustically by a human operator, or by the vehicle's autonomy system.



End